Sec 2.1: Statements

Mathematics is the business of proving mathematical statements to be true or false. Logic lays the foundation for rigorous mathematical proofs.

Definition: A **statement** is a sentence that is either true or not.

Give examples of some statements.

Give an example of a sentence which is **not a statement**.

"This statement is false."

The above is an example of a self-referential sentence.
Determine if the following are statements. Explain.

1. "Assume that the set A is nonempty."

2. "The set A is nonempty."

3. "The set A, defined by $A = \{x \in \mathbb{R} \quad | \quad x^2 + 5 = 0\}$ is nonempty."

4. $P(x) : x^2 - 8 \geq 0$.

Be pedantic!

Which of these statements are true?

(1) There are 18 students registered for this class.

(2) There are 5 students registered for this class.

(3) There are 50 students registered for this class.

(4) There is a student registered for this class.

(5) There are no students registered for this class.

Sec 2.2 : The Negation of a Statement

Definition: The **negation** of statement A is another statement that is interpreted as being false when A is true and true when A is false.

The negation of the statement A is written as $\sim A$ or not (A).

- A: "I like ice cream."

  $\sim A$:

  $\sim (\sim A)$:

- B: "All sheep are black."

  $\sim B$:

  $\sim (\sim B)$:

Theorem: $\sim(\sim A)$ is equivalent to A.

Truth Tables (Sec 2.8 : Logical Equivalence)

A	$\sim A$	$\sim(\sim A)$
T		
F		

Definition: Two statements P and Q are called logically equivalent if the two statements have the same truth values for *all combinations of truth values of their component statements.*

Notation: If two statement P and Q are logically equivalent, then this is denoted by $P \equiv Q$

Remark: If we can show that P is true, then Q is true as well.

1. Statements with AND ($A \wedge B$)

"Yesterday I went biking and I saw a fox."

If this statement is not true, what must be true?
(What is the negation of the above?)

Definition: $A \wedge B$ is true only if both A and B are true.

A	B	A and B	not(A and B)
T	T		
T	F		
F	T		
F	F		

2. Statements with OR ($A \vee B$)

"I have a candy in my left pocket or in my right pocket."

$x \in A \cup B$ is equivalent to $x \in A$ or $x \in B$.

Definition: $A \vee B$ is true when at least one of A or B is true.

The mathematical OR is not exclusive.
Unlike the conversational OR, it is not "either - or"!

List some statements with OR

A	B	$A \vee B$	$\sim (A \vee B)$
T	T		
T	F		
F	T		
F	F		

3. Negation of and statement

- A: "Jesse is tall" B: "Daniel is tall"
- A $\wedge$ B :
- $\sim$(A $\wedge$ B) :

Theorem: The **negation** of A and B is equivalent to (not A) or (not B).

4. Negations of or statements

- A: "Rachel's major is mathematics" B: "Asia's major is mathematics"
- A $\vee$ B :
- $\sim$(A $\vee$ B) :

Theorem: The **negation** of A or B is equivalent to (not A) and (not B).

5. Negation of AND and OR

A	B	$\sim A$	$\sim B$	$A \vee B$	$\sim (A \vee B)$	$(\sim A) \wedge (\sim B)$
T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			

Theorem: not(A or B) is equivalent to not(A) and not(B).

$$\sim (A \vee B) \equiv (\sim A) \wedge (\sim B)$$

Prove on your own:

Theorem: not(A and B) is equivalent to not(A) or not(B).

$$\sim (A \wedge B) \equiv (\sim A) \vee (\sim B)$$