Math 320-002 **Review for Exam 1** **Spring, 2017**

I. Be able to state important definitions and theorems.

II. Review homework problems.

III. Review quizzes.

IV. Be able to prove short and straightforward theorems (e.g. problems 1, 2, 6, and 8).

Important definitions you should be able to state

1. Upper bound, lower bound, min, max, supremum, infimum for a set in $\mathbb{R}$.
2. What do we mean by $\lim_{n\to\infty} s_n = L$, $\lim_{n\to\infty} s_n = +\infty$, $\lim_{n\to\infty} s_n = -\infty$?
3. Increasing, decreasing sequence.
4. Definition of limsup and liminf.
5. Cauchy sequence.

Important theorems/axioms you should be able to state/prove

1. Proving basic properties of real numbers based on given axioms.
2. Completeness Axiom (state)
3. Proving basic properties of inf and sup. (See problems 4.5-4.9, 4.14, 4.16)
4. Proving a given sequence converges using the definition. (See Sect. 8)
5. Be able to state and apply the Squeeze Theorem
6. Prove: Convergent sequences are bounded. (See Sect. 9)
7. Prove: If (s_n) and (t_n) converge, for $a, b \in \mathbb{R}$, prove $\lim_{n\to\infty}(as_n + bt_n) = a \lim_{n\to\infty} s_n + b \lim_{n\to\infty} t_n$. (See Sect. 9)
8. Proving a given sequence converges using limit theorems (See Sect. 9)
9. Prove: Given $s_n > 0$ for all $n \in \mathbb{N}$, prove $\lim_{n\to\infty} s_n = +\infty$ if and only if $\lim_{n\to\infty} \frac{1}{s_n} = 0$. (See Sect. 9)
10. Be able to use: All bounded monotone sequences converge. (See problems 10.9-10.12)
11. Be able to prove: If (s_n) is unbounded increasing sequence, then $\lim_{n\to\infty} s_n = +\infty$. (Th. 10.4)
12. Prove: If a sequence is convergent, then it is Cauchy. (Sect 10)

In addition, these problems might help you review the material.

1. Let A be a nonempty subset of $\mathbb{R}$ that is bounded above, and $B = \{a + 7 : a \in A\}$. Prove that B is also bounded above and $\sup B = 7 + \sup A$.

2. Assume S and T are nonempty bounded sets. Prove that if $S \subseteq T$, then $\inf(T) \leq \inf(S)$.

3. Use the definition of convergence to prove $\lim_{n\to\infty} \frac{3n^3 + 7n + 1}{n^3 - n - 3} = 3$.

4. Let $a_n = 5 + 3(-1)^n$ for $n \in \mathbb{N}$. Prove (a_n) diverges.

5. Let $a_1 = 1$ and $a_{n+1} = \frac{1}{5}(3a_n + 1)$ for $n \geq 1$. Prove $\lim_{n\to\infty} a_n$ exists and find the limit.

6. Use the definition to prove that if $\lim_{n\to\infty} a_n = -\infty$ and $\lim_{n\to\infty} b_n = 7$, then $\lim_{n\to\infty}(2a_n + 3b_n) = -\infty$.

7. Use the appropriate limit theorems (you do not need to start from the definition) to prove the following sequences converge.

   (a) $a_n = \sqrt{n^2 + 1} - n, n \in \mathbb{N}$.

   (b) $b_n = \frac{\sin(3n) + 7}{n}, n \in \mathbb{N}$.

   (c) $c_n = \frac{n^5 - 10n^2 + 100n - 1}{n^6 - 8n^5 + \pi}, n \in \mathbb{N}$.

8. Prove that if $a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} a_n = +\infty$, then $\lim_{n\to\infty} b_n = +\infty$.