

Name \_\_\_\_\_

*Problem 1.* Let  $A = \{5k : k \in \mathbb{N}\}$ . Prove  $|A| = |\mathbb{N}|$  by constructing an explicit bijection between  $A$  and  $\mathbb{N}$  and proving that it is indeed a bijection.

*Problem 2.* Assume  $A$  and  $B$  are two non-empty sets such that  $A \cap B = \emptyset$ . Assume  $|A| = n$  for some  $n \in \mathbb{N}$  and  $|B| = |\mathbb{N}|$ . Prove that  $|A \cup B| = |\mathbb{N}|$ .

For this problem specify an explicit bijection between  $A \cup B$  and  $\mathbb{N}$ , but you do not need to prove it is a bijection in a rigorous way.

*Problem 3.* Assume  $A$  and  $B$  are two non-empty sets such that  $A \cap B = \emptyset$ . Assume  $|A| = |\mathbb{N}|$  and  $|B| = |\mathbb{N}|$ . Prove that  $|A \cup B| = |\mathbb{N}|$ .

For this problem specify an explicit bijection between  $A \cup B$  and  $\mathbb{N}$ , but you do not need to prove it is a bijection in a rigorous way. Looking at the proof we did in class for  $|\mathbb{N}| = |\mathbb{Z}|$  might be useful.