Math 421-001 **Quiz #2** **Jan. 23, 2015**

Problem 1. *(15 points)* Let $S = \{2, 3\}$ and $f : [0, 5] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 4, & x \in S \\ 2, & x \in [0,5] \setminus S. \end{cases}$$

Prove that f is Riemann integrable on $[0, 5]$.

Solution: We need to show that for every positive real number ε there exists a partition P of $[0, 5]$ such that $U(f, P) - L(f, P) < \varepsilon$. Here $U(f, P)$ and $L(f, P)$ denote respectively the upper and lower Riemann sums of f corresponding to P.

Let $\varepsilon > 0$ be given. Let $P = \{0, 2 - \delta, 2 + \delta, 3 - \delta, 3 + \delta, 5\}$, where $\delta = \min\{\frac{\varepsilon}{10}, 1\}$. Then

$$U(f,P)-L(f,P) = (2-2)\triangle x_1+(4-2)\triangle x_2+(2-2)\triangle x_3+(4-2)\triangle x_4+(2-2)\triangle x_5 = 8\delta \le \frac{4}{5}\varepsilon < \varepsilon.$$

Here $\triangle x_i$ denotes the length of the i^{th} interval in the partition P.

Thus, for every $\varepsilon > 0$ there exists a partition P of $[0, 5]$, such that $U(f, P) - L(f, P) < \varepsilon$, i.e., f is Riemann integrable on $[0, 5]$.