Securitization of Longevity Risk in Reverse Mortgages

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Securitization of longevity risks

- We are witnessing a dramatic improvement in mortality across the world.
- Annuity and pension providers are primarily affected by this longevity risk:
  - annuity benefits are usually paid longer when there is a longer life expectancy
- Securitization is a vehicle to re-package the risk so that it may be transferred to the financial market:
  - essentially re-packaging the cashflows into securities that can be traded
  - survivor or mortality bonds; survivor swaps - mortality derivatives

Some advantages to securitization

- Significantly larger capacity of the capital market (vis-a-vis the insurance market)
- Improved pricing as there are larger number of investors (against a limited number of insurance providers)
- Helps investors to diversify since many believe insurance risk may be considered uncorrelated to many asset classes
- Reduces counterparty default risk
- Lowers the cost of capital

Some research works published

- Pricing for such securitized instruments can be tricky:
  - mortality improvement model or forecasting of mortality: extension of Lee-Carter, term structure of mortality
  - valuation issues related to incomplete markets
- Some useful early works:
  - Blake and Burrows (2001)
  - Cowley and Cummins (2005); Cairns, Blake and Dowd (2006)
  - Lin and Cox (2005); Cox, Lin and Wang (2006)
  - Liao, Yang and Huang (2007); Wills and Sherris (2010)

Reverse mortgages

A reverse mortgage is a special type of loan that allows you to convert a proportion of your home equity into cash:

- lump sum, annuity, line of credit or a combination

There are special features that make it different from a conventional loan:

- no repayments of principal or interest, but outstanding balance still accrues with interest
- repayment is when you as borrower dies, or voluntarily leaves the property
- you can continue to live in the property
- "nonrecourse" where lender can never recover from other assets of borrower
- underwriting is usually based solely on the value of the home

Risks of reverse mortgages to lenders

Although there are benefits associated with reverse mortgages, they are not without the presence of risks:

- occupancy risk and longevity risk
- interest rate risk
- house price risk
- other risks: maintenance, expenses
  - usually associated with inflation

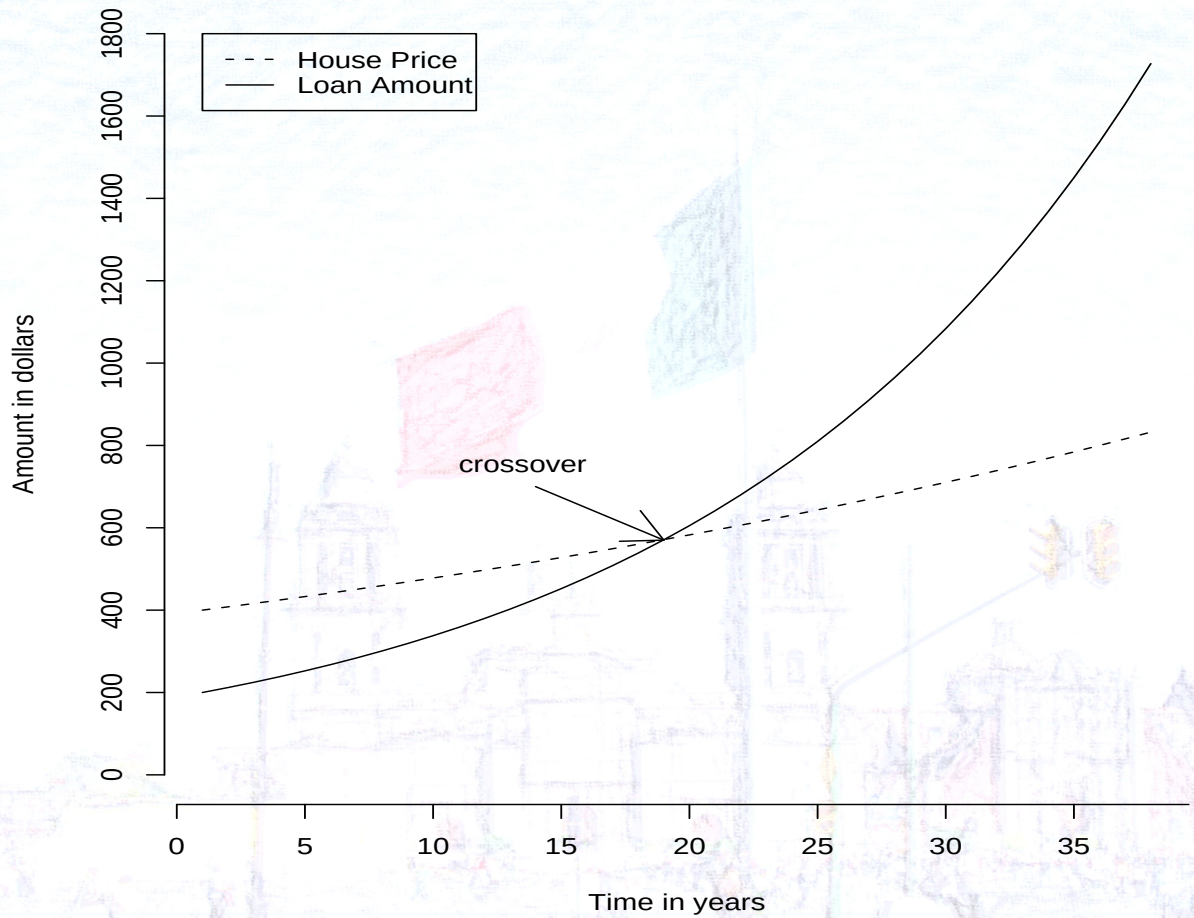

Figure : Illustration of the crossover risk

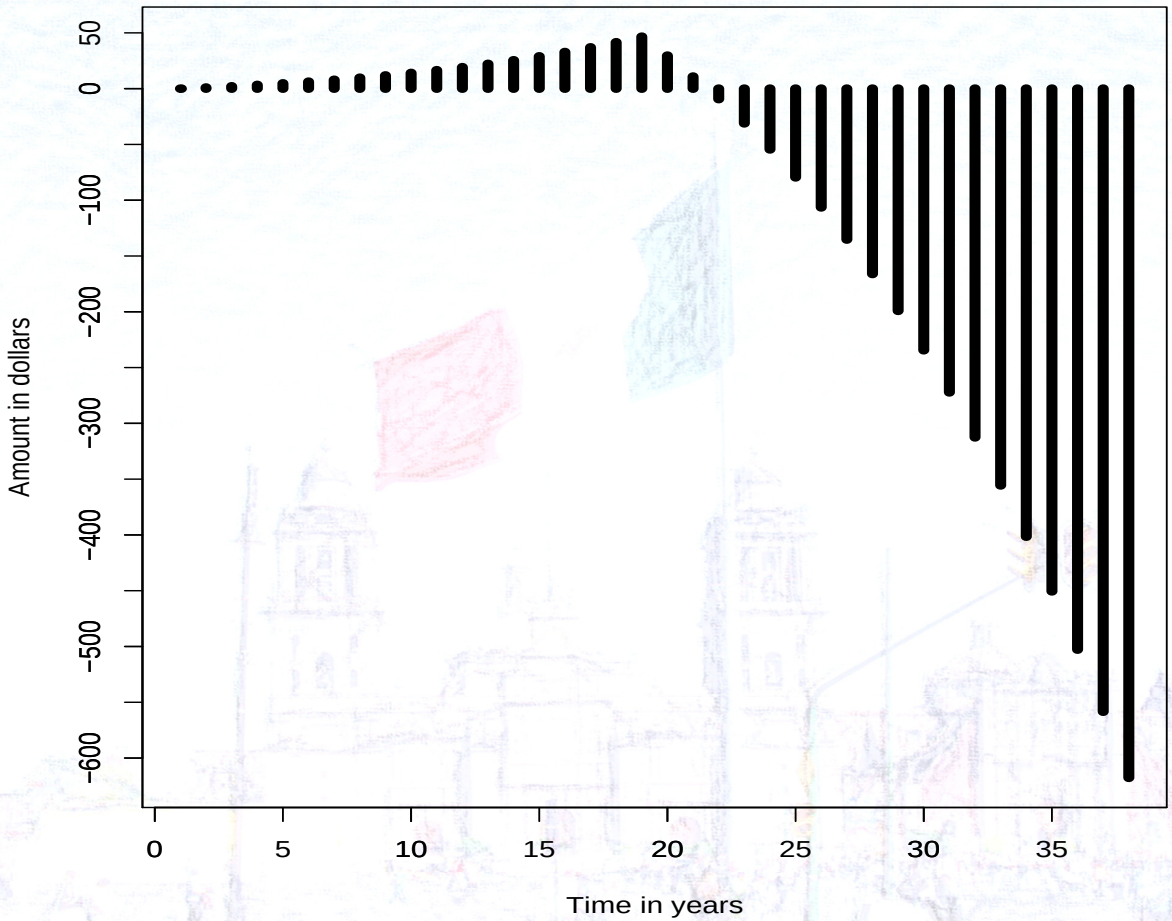

Figure : Cash flow analysis for reverse mortgage lender

Pricing lump sum reverse mortgages

The pricing equation for a lump sum reverse mortgage can be expressed as:

$$\mathsf{E}\left[e^{-rT}Q_0\exp\left(\int_0^T \eta_s ds\right)\right]$$
$$=\mathsf{E}\left[e^{-rT}\min\left[Q_0\exp\left(\int_0^T (r_s+\lambda)\,ds\right), H_0\exp\left(\int_0^T \delta_s ds\right)\right]\right]$$

where:

- Q_0 is the initial loan amount
- cost of capital accumulates at η
- H_0 is the initial house price
- house appreciation rate is δ
- λ is the actuarially fair risk premium

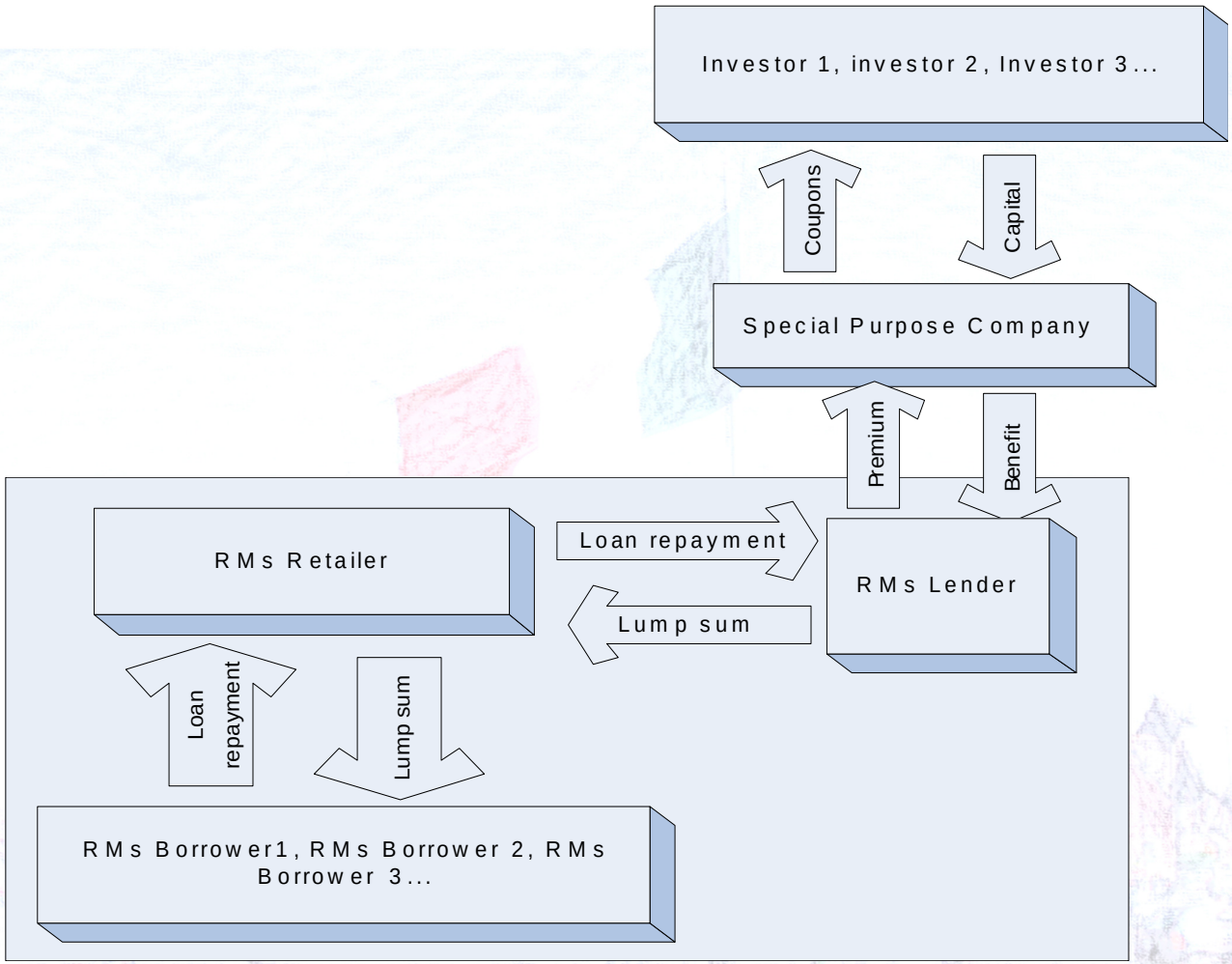

Figure : Recommended structure of the reverse mortgage securitization

Cash flow analysis for each component

- Retailer
- Lender
- SPC (special purpose company)
- Investors

Reverse mortgage survivor bond Type 1

Suppose lender holds a portfolio of l_0 loans. At time 0, assume all borrowers are of the same age, and each borrow a lump sum of Q_0 against their home currently valued at H_0.

To hedge longevity risk, lender purchases insurance from the SPC at a lump sum premium of P.

Here, in each period after the crossover, SPC will pay the lender a benefit of $A_t\left(l_t - \widehat{l_t}\right)$, up to a ceiling amount of C, if the number of survived loans l_t exceeds the predetermined trigger $\widehat{l_t}$.

At t, loss amount for each loan i is $L_{i,t}$, and since interest and house appreciation rates are constant, $L_{i,t} = L_t$ for all i and t.

The amount A_t is determined as

$$A_t = \frac{L_{t+1}}{1+r} - L_t.$$

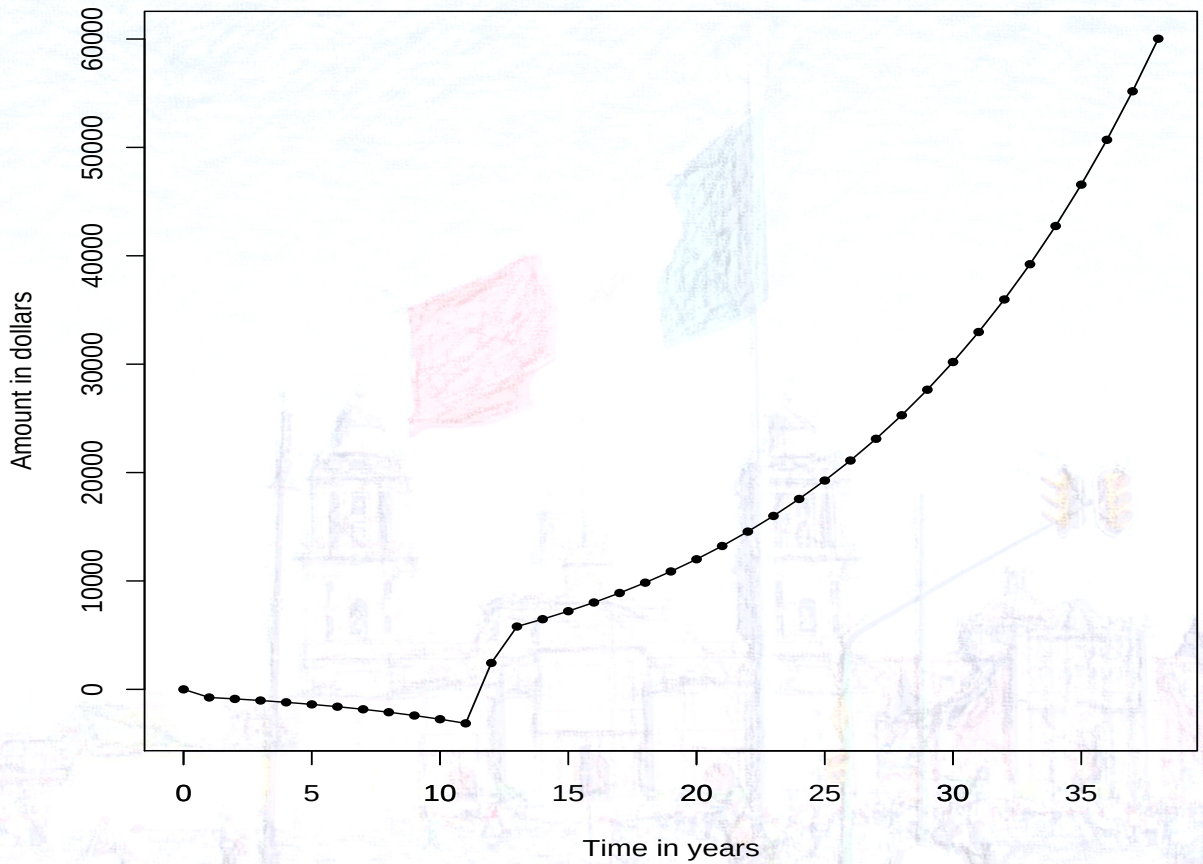

Figure : Single loss L_t in each period

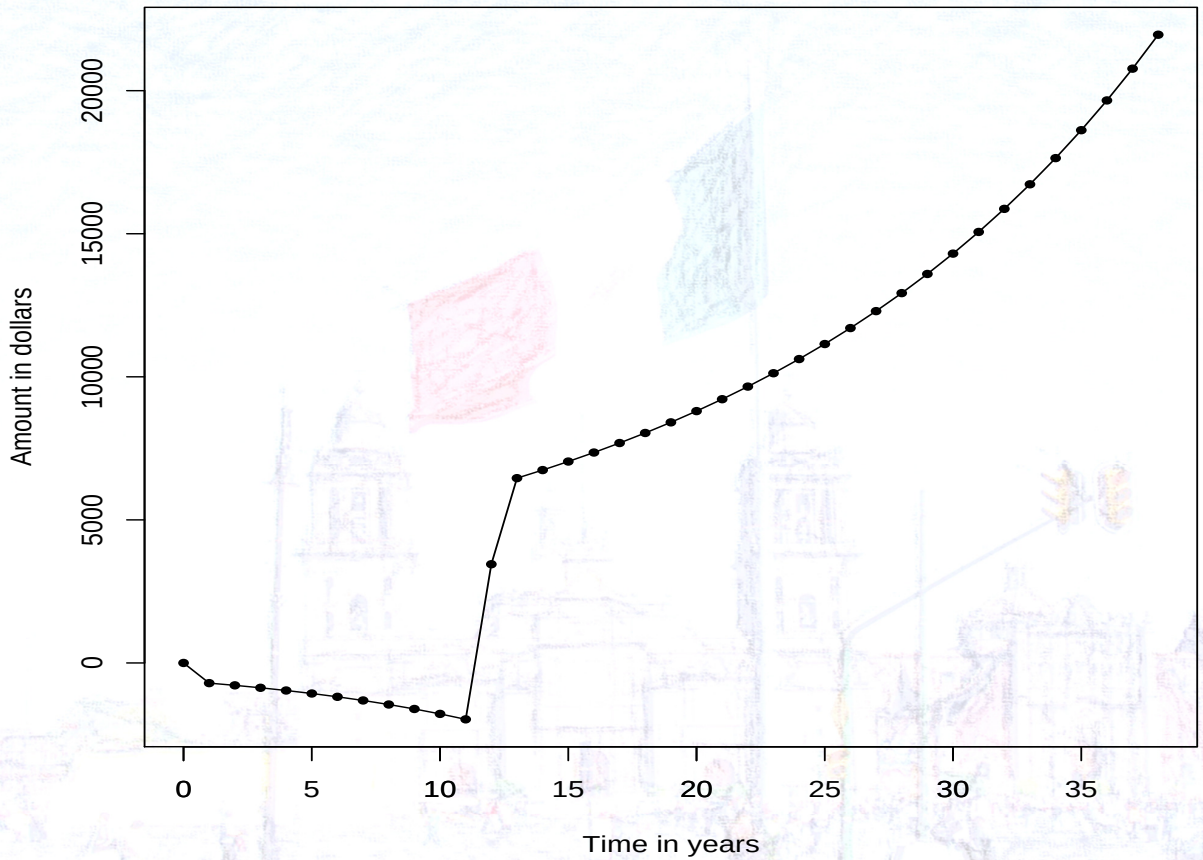

Figure : Appreciation A_t of each loss in each period

Pricing for survivor bond Type 1

Suppose a series of $\widehat{l}_t$ are determined as triggers, for a portfolio of l_0 loans borrowed by persons aged x with the identical house value H_0, the bondholders will receive coupons at t

$$C_t = \begin{cases} C & \text{if } \; l_t \leq \widehat{l}_t \\ C - A_t\left(l_t - \widehat{l}_t\right) & \text{if } \; \widehat{l}_t < l_t < \frac{C}{A_t} \\ 0 & \text{if } \; l_t > \frac{C}{A_t} \end{cases} .$$

This is equivalent to

$$C_t = C - \left[A_t\left(l_t - \widehat{l}_t\right), 0\right]_+ + \left[A_t\left(l_t - \widehat{l}_t\right) - C, 0\right]_+ .$$

The pricing equation of the survivor bond type 1:

$$V = Fv^T + \sum_{k=1}^{T} v^k \left\{ C - \mathsf{E}\left[A_t\left(l_t - \widehat{l}_t\right), 0\right]_+ + \mathsf{E}\left[A_t\left(l_t - \widehat{l}_t\right) - C, 0\right]_+ \right\}.$$

Numerical illustration

For purposes of illustration, let the annual interest rate r be 6.5%, annual house price appreciation c be 3%, the risk premium the lender charges λ_1 and is charged λ_2 be 3% and 1.5%, respectively, and the house price H_0 be $\$100,000$.

Based on some initial simulations, the initial loan amount Q_0 is set at \$39,222 which gives the maximal safe loan amount.

Then, the L_t and A_t in each period are shown in the subsequent tables.

Single loss in each period

Period	Loss L_t	Period	Loss L_t	Period	Loss L_t
1	−588.33	14	−3590.22	27	16738.21
2	−691.29	15	−4059.30	28	18400.75
3	−807.79	16	2500.51	29	20206.01
4	−939.43	17	5935.67	30	22165.68
5	−1087.97	18	6651.23	31	24292.41
6	−1255.36	19	7431.25	32	26599.90
7	−1443.77	20	8281.12	33	29102.90
8	−1655.62	21	9206.63	34	31817.40
9	−1893.57	22	10214.08	32	34760.64
10	−2160.57	23	11310.25	36	37951.27
11	−2459.91	24	12502.48	37	41409.46
12	−2795.21	25	13798.72	38	45156.95
13	−3170.50	26	15207.54		

Appreciation of each loss in each period

Period	Appreciation A_t	Period	Appreciation A_t	Period	Appreciation A_t
1	−552.42	14	−2092.50	27	11173.30
2	−613.19	15	−2313.83	28	11712.80
3	−680.39	16	4093.37	29	12284.82
4	−754.69	17	7166.25	30	12891.65
5	−836.83	18	7475.87	31	13535.76
6	−927.60	19	7802.34	32	14219.77
7	−1027.90	20	8146.79	33	14946.54
8	−1138.70	21	8510.39	34	15719.13
9	−1261.07	22	8894.45	32	16540.83
10	−1396.21	23	9300.32	36	17415.19
11	−1545.42	24	9729.49	37	18346.04
12	−1710.12	25	10183.56	38	19337.48
13	−1891.91	26	10664.21		

Projected trigger values in each period

Period	Trigger $\widehat{l}_t$	Period	Trigger $\widehat{l}_t$	Period	Trigger $\widehat{l}_t$
1	986	14	677	27	152
2	973	15	639	28	124
3	958	16	599	29	99
4	942	17	557	30	79
5	924	18	514	31	61
6	904	19	470	32	48
7	883	20	427	33	37
8	859	21	384	34	28
9	834	22	342	32	21
10	807	23	300	36	15
11	778	24	259	37	11
12	746	25	220	38	8
13	712	26	183		

Calculation of mortality bond price (Type 1)

Number of loans	1000
Initial house value	$\$100,000$
Lump sum borrowed	$\$39,222$
Face value of straight bond	$\$100,000,000$
Face value of survivor bond	$\$100,000,000$
Coupon rate for both bonds	6.5% p.a.
Annual aggregate cash flow out of SPC	$\$6,500,000$
Straight bond price	$\$100,000,000$
Survivor bond price	$\$99,902,898$
Premium paid to SPC	$\$97,102$

Results of sensitivity testing (Type 1)

Shock ε_t	Statistic	l_{20}	PV of coupons and principal	Percentage change
1%	Min	429	99,790,111	−0.11%
	5% percentile	430	99,793,980	−0.11%
	95% percentile	432	99,795,946	−0.11%
	Max	435	99,796,337	−0.11%
	Mean	431	99,795,199	−0.11%
	Stdev	1	635	
5%	Min	435	99,768,435	−0.13%
	5% percentile	440	99,785,248	−0.12%
	95% percentile	453	99,795,809	−0.11%
	Max	465	99,797,493	−0.11%
	Mean	446	99,791,922	−0.11%
	Stdev	4	3,369	
10%	Min	444	99,744,368	−0.16%
	5% percentile	453	99,773,260	−0.13%
	95% percentile	481	99,798,092	−0.11%
	Max	504	99,786,999	−0.10%
	Mean	465	99,786,940	−0.12%
	Stdev	8	6,946	
25%	Min	465	99,663,820	−0.24%
	5% percentile	491	99,714,999	−0.19%
	95% percentile	569	99,783,484	−0.12%
	Max	641	99,793,361	−0.11%
	Mean	528	99,758,254	−0.14%
	Stdev	24	21,573	
50%	Min	411	99,482,588	−0.42%
	5% percentile	483	99,557,089	−0.35%
	95% percentile	699	99,751,549	−0.15%
	Max	825	99,774,291	−0.13%
	Mean	584	99,663,909	−0.24%
	Stdev	65	69,833	

Concluding remarks

- The reverse mortgage is a promising financial product with many potential economic benefits to both consumers and suppliers.
- However, due to the various risks involved in reverse mortgages, especially the longevity risk component, the development of the product has to some extent been stunted.
- We suggest using securitization to deal with the risks to the lender, particularly the longevity risk component.
  - Our results indicate that mortality securitization is a good method to manage longevity risk in reverse mortgages.
  - Given the many benefits of mortality securitization, we believe that it can help the future development of reverse mortgage products in the capital market.

Main reference

Wang, L., Valdez, E.A., and Piggott, J., 2008, "Securitization of Longevity Risk in Reverse Mortgages", *North American Actuarial Journal*, Vol 12, No 4, pp. 345-371.

- Thank you -