Please follow the instructions below:

Return this page with your signature.

Submit your work to our graduate assistant, Ed Cruz, at C505 Wells.

Write your name and section number at the spaces provided below.

Good luck on your finals. Enjoy the holidays.

Name: _____   **Section:** _____

I certify that this is my own work, and that I have not copied the work of another student.

Signature: _____   **Date:** _____

1. (30 points) For a special life insurance policy issued to (45), you are given:

   - A death benefit of 100,000 is paid at the end of the year of death, if death occurs after age 60.
   - There are no benefits paid, if death occurs before age 60.
   - Premiums are payable monthly, at the beginning of each month, until reaching age 60.

   Premiums are calculated based on the following assumptions:

   - Mortality follows the `Illustrative Life Table`.
   - $i = 0.06$
   - The Woolhouse approximate formula with two terms is used to calculate annuity values.

   (a) [5 points] Calculate the actuarial present value of the death benefit.

   (b) [10 points] Use the equivalence principle to calculate the monthly benefit premium for this policy. Give the annualized benefit premium as well.

   (c) [10 points] Suppose premiums are paid annually, at the beginning of each year, until reaching age 60. Calculate the annual benefit premium.

   (d) [5 points] Explain why there is a difference between the annualized premium calculated in (b) and the annual premium calculated in (c).

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

2. (35 points) For a group of 200 lives with select age $[x]$ and with independent future lifetimes, you are given:

- Each life is to be paid \$1 at the beginning of each year, if alive.

- $A_{[x]} = 0.20$

- $^2A_{[x]} = 0.06$

- $i = 0.04$

Y_i is the present value random variable of the payments for life i, $i = 1, 2, \ldots, 200$, so that the present value of the aggregate payments is the sum of these:

$$Y = \sum_{i=1}^{200} Y_i$$

(a) [5 points] Calculate $E[Y]$.

(b) [10 points] Calculate $\text{Var}[Y]$.

(c) [15 points] Using the Normal approximation to Y, calculate the initial size of the fund needed in order to be 95% certain of being able to make the payments for these life annuities.

(d) [5 points] If you need an initial fund that ensures you 90% certain of being able to cover the payments, would you expect this to be higher or lower than that in (c)? Explain why. Do not do any calculations.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

3. (35 points) You are given the following survival function for a newborn:

$$S_0(x) = \frac{1}{c}(110 - x)^{2/3},$$

for some constant c.

(a) [5 points] Calculate c so that this survival function is legitimate, and give the limiting age for this survival model.

(b) [5 points] Calculate the expected future lifetime of a newborn.

(c) [5 points] Calculate the probability that a newborn will survive to reach his expected future lifetime.

(d) [5 points] Calculate the probability that a newborn will reach to age 65 but die within 20 years following that.

A special 3-year endowment insurance policy is issued to (50) where:

- The death benefit of 200 is payable at the end of the year of death.
- The endowment is 500 payable at maturity.

(e) [15 points] Assuming this survival model and $i = 0.05$, calculate the actuarial present value of this insurance policy.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000{}_5E_x$	$1000{}_{10}E_x$	$1000{}_{20}E_x$	x
0	10,000,000	20.42	16.8010	49.00	25.92	728.54	541.95	299.89	0
5	9,749,503	0.98	17.0379	35.59	8.45	743.89	553.48	305.90	5
10	9,705,588	0.85	16.9119	42.72	9.37	744.04	553.34	305.24	10
15	9,663,731	0.91	16.7384	52.55	11.33	743.71	552.69	303.96	15
20	9,617,802	1.03	16.5133	65.28	14.30	743.16	551.64	301.93	20
21	9,607,896	1.06	16.4611	68.24	15.06	743.01	551.36	301.40	21
22	9,597,695	1.10	16.4061	71.35	15.87	742.86	551.06	300.82	22
23	9,587,169	1.13	16.3484	74.62	16.76	742.68	550.73	300.19	23
24	9,576,288	1.18	16.2878	78.05	17.71	742.49	550.36	299.49	24
25	9,565,017	1.22	16.2242	81.65	18.75	742.29	549.97	298.73	25
26	9,553,319	1.27	16.1574	85.43	19.87	742.06	549.53	297.90	26
27	9,541,153	1.33	16.0873	89.40	21.07	741.81	549.05	297.00	27
28	9,528,475	1.39	16.0139	93.56	22.38	741.54	548.53	296.01	28
29	9,515,235	1.46	15.9368	97.92	23.79	741.24	547.96	294.92	29
30	9,501,381	1.53	15.8561	102.48	25.31	740.91	547.33	293.74	30
31	9,486,854	1.61	15.7716	107.27	26.95	740.55	546.65	292.45	31
32	9,471,591	1.70	15.6831	112.28	28.72	740.16	545.90	291.04	32
33	9,455,522	1.79	15.5906	117.51	30.63	739.72	545.07	289.50	33
34	9,438,571	1.90	15.4938	122.99	32.68	739.25	544.17	287.82	34
35	9,420,657	2.01	15.3926	128.72	34.88	738.73	543.18	286.00	35
36	9,401,688	2.14	15.2870	134.70	37.26	738.16	542.11	284.00	36
37	9,381,566	2.28	15.1767	140.94	39.81	737.54	540.92	281.84	37
38	9,360,184	2.43	15.0616	147.46	42.55	736.86	539.63	279.48	38
39	9,337,427	2.60	14.9416	154.25	45.48	736.11	538.22	276.92	39
40	9,313,166	2.78	14.8166	161.32	48.63	735.29	536.67	274.14	40
41	9,287,264	2.98	14.6864	168.69	52.01	734.40	534.99	271.12	41
42	9,259,571	3.20	14.5510	176.36	55.62	733.42	533.14	267.85	42
43	9,229,925	3.44	14.4102	184.33	59.48	732.34	531.12	264.31	43
44	9,198,149	3.71	14.2639	192.61	63.61	731.17	528.92	260.48	44
45	9,164,051	4.00	14.1121	201.20	68.02	729.88	526.52	256.34	45
46	9,127,426	4.31	13.9546	210.12	72.72	728.47	523.89	251.88	46
47	9,088,049	4.66	13.7914	219.36	77.73	726.93	521.03	247.08	47
48	9,045,679	5.04	13.6224	228.92	83.06	725.24	517.91	241.93	48
49	9,000,057	5.46	13.4475	238.82	88.73	723.39	514.51	236.39	49
50	8,950,901	5.92	13.2668	249.05	94.76	721.37	510.81	230.47	50
51	8,897,913	6.42	13.0803	259.61	101.15	719.17	506.78	224.15	51
52	8,840,770	6.97	12.8879	270.50	107.92	716.76	502.40	217.42	52
53	8,779,128	7.58	12.6896	281.72	115.09	714.12	497.64	210.27	53
54	8,712,621	8.24	12.4856	293.27	122.67	711.24	492.47	202.70	54
55	8,640,861	8.96	12.2758	305.14	130.67	708.10	486.86	194.72	55
56	8,563,435	9.75	12.0604	317.33	139.11	704.67	480.79	186.32	56
57	8,479,908	10.62	11.8395	329.84	147.99	700.93	474.22	177.53	57
58	8,389,826	11.58	11.6133	342.65	157.33	696.85	467.12	168.37	58
59	8,292,713	12.62	11.3818	355.75	167.13	692.41	459.46	158.87	59
60	8,188,074	13.76	11.1454	369.13	177.41	687.56	451.20	149.06	60
61	8,075,403	15.01	10.9041	382.79	188.17	682.29	442.31	139.00	61
62	7,954,179	16.38	10.6584	396.70	199.41	676.56	432.77	128.75	62
63	7,823,879	17.88	10.4084	410.85	211.13	670.33	422.54	118.38	63
64	7,683,979	19.52	10.1544	425.22	223.34	663.56	411.61	107.97	64
65	7,533,964	21.32	9.8969	439.80	236.03	656.23	399.94	97.60	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	1000_5E_x	$1000_{10}E_x$	$1000_{20}E_x$	x
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90
91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110