MATH 3630
Actuarial Mathematics I
Class Test 1
Wednesday, 5 October 2011
Time Allowed: 1 hour
Total Marks: 100 points
Please write your name and student number at the spaces provided:

Name: $\qquad$ Student ID:

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:
For a standard life, the force of mortality is

$$
\mu_{x}=\frac{1}{2(100-x)}, \text { for } 0 \leq x<100
$$

For a substandard life, the force of mortality is twice that of a standard life of the same age. Calculate the probability that a substandard life currently age 50 will survive to reach age 65 .

## Question No. 2:

Suppose you are given the following select-and-ultimate mortality table:

| $[x]$ | $\ell_{[x]}$ | $\ell_{[x]+1}$ | $\ell_{[x]+2}$ | $\ell_{x+3}$ | $\mathrm{x}+3$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 55 | 882 | 877 | 871 | 864 | 58 |
| 56 | 875 | 870 | 863 | 856 | 59 |
| 57 | 868 | 863 | 856 | 849 | 60 |
| 58 | 861 | 855 | 848 | 840 | 61 |
| 59 | 854 | 847 | 840 | 832 | 62 |
| 60 | 846 | 839 | 832 | 823 | 63 |

Calculate the probability that a life now age 56 , with select age 55 , will die between the ages of 57 and 61 .

## Question No. 3:

Assume mortality follows the Illustrative Life Table.
Suppose that the constant force assumption holds between integral ages.
Calculate ${ }_{2 \mid 1.5} q_{40.25}$ and interpret this probability.

Question No. 4:
You are given:

- $\stackrel{\circ}{e}_{30}=51.50, \quad \stackrel{\circ}{e}_{35}=46.68, \quad$ and $\quad \stackrel{\circ}{e}_{40}=41.91$
- $\dot{e}_{30: 51}=4.988$ and $\stackrel{\circ}{30: 10 \mid}=9.963$

Calculate ${ }_{5} p_{35}$.

Question No. 5:
You are given:

$$
\mu_{x}= \begin{cases}0.04, & 0<x<10 \\ 0.08, & x \geq 10\end{cases}
$$

Calculate ${ }_{10} p_{5}$.

## Question No. 6:

For a life $(x)$, you are given $\ell_{x}=1000$ and the following extract from a mortality table:

| $k$ | $d_{x+k}$ |
| :---: | :---: |
| 0 | 400 |
| 1 | 200 |
| 2 | 200 |
| 3 | 200 |

Calculate the variance of $K_{x}$.

## Question No. 7:

For a population which consists of $60 \%$ males (m) and $40 \%$ females (f) at birth, you are given:

| $x$ | $S_{0}^{\mathrm{m}}(x)$ | $S_{0}^{\mathrm{f}}(x)$ |
| :---: | :---: | :---: |
| 50 | 0.08 | 0.10 |
| 51 | 0.07 | 0.09 |

Calculate $q_{50}$ for a randomly chosen individual from this population.

## Question No. 8:

Suppose you are given:

$$
S_{0}(x)=1-\left(\frac{x}{100}\right)^{2}, \quad \text { for } 0 \leq x \leq 100
$$

Calculate $\mu_{50}$.

## Question No. 9:

You are given the following extract from a select-and-ultimate mortality table:

| $[x]$ | $q_{[x]}$ | $q_{[x]+1}$ | $q_{[x]+2}$ | $q_{x+3}$ | $x+3$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 62 | 0.11 | 0.13 | 0.15 | 0.17 | 65 |
| 63 | 0.12 | 0.14 | 0.16 | 0.18 | 66 |
| 64 | 0.13 | 0.15 | 0.17 | 0.19 | 67 |

Calculate the probability that a life with select age 62 will survive the next three years.

Question No. 10:
You are given:

- $\ell_{x}=800$
- $\ell_{x+0.5}=700$
- $\ell_{x+1}=600$

Using trapezium (trapezoidal) rule with $h=0.5$, estimate the one-year temporary expectation of life of $(x)$.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

