MATH 3630
Actuarial Mathematics I
Class Test 2
Friday, 14 November 2008
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTIONS Student ID: EMIL

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

Let T_x denote the future lifetime random variable for (x). You are given:

- T_x has an exponential distribution with parameter μ.
- Force of interest is constant at δ.
- $\bar{A}_x = 0.4118$.

Calculate ${}^2\bar{A}_x$.

$T_x \sim$ Exponential $\Rightarrow$ constant μ

$$\therefore \bar{A}_x = \frac{\mu}{\mu+\delta} = 0.4118 \Rightarrow (1-.4118)\mu = .4118\,\delta$$

$$\delta = (.5882/.4118)\mu$$

$${}^2\bar{A}_x = \frac{\mu}{\mu+2\delta} = \frac{\mu}{\mu+2(.5882/.4118)\mu} = \frac{1}{1+2(.5882/.4118)}$$

$$= \underline{\underline{0.259287}}$$

Question No. 2:

You are given:

x	q_x	$\ddot{a}_x$
75	.03814	7.4927
76	.04196	7.2226

Calculate the interest rate i.

Using recursion, $\ddot{a}_{75} = 1 + v\, p_{75}\, \ddot{a}_{76}$

$$7.4927 = 1 + v(1 - .03814)(7.2226)$$

$$\Rightarrow \quad v = \frac{6.4927}{.96186(7.2226)} = 0.93458737$$

$$\Rightarrow \quad i = \frac{1}{0.93458737} - 1 = \underline{7\%}$$

Question No. 3:

For a continuous whole life annuity of 1 on (x), you are given that:

- T_x, the future lifetime, has a constant force of mortality of 0.06;
- the force of interest is also constant at 4%.

Calculate $P\left(\bar{a}_{\overline{T_x}|} > \bar{a}_x\right)$. Interpret this probability.

Constant forces $\Rightarrow$ $\bar{a}_x = \dfrac{1}{\mu+\delta} = \dfrac{1}{.06+.04} = \dfrac{1}{.10}$

$= 10$

$T_x \sim \text{Exponential}(\mu = 0.06)$

$\therefore P\left(\bar{a}_{\overline{T_x}|} > 10\right) = P\left(\dfrac{1-v^{T_x}}{\delta} > 10\right)$

$= P\left(T_x > \dfrac{\log(1-10\delta)}{-\delta}\right)$

$= P\left(T_x > \dfrac{\log(.6)}{-.04} = 12.7706406\right)$

$= e^{-.06(12.7706406)}$

$= \underline{\underline{.464758}}$ refers to the probability that the P.V. of the annuity will exceed the APV of the annuity (or the expected PV of the annuity)

Question No. 4:

For a group of 25 individuals all age x, you are given:

- their future lifetimes are independent;
- each individual is paid 10 at the beginning of each year, if alive;
- $A_x = 0.369131$;
- ${}^2A_x = 0.1774113$; and
- $i = 6\%$.

Using Normal approximation, calculate the size of the fund needed at inception in order to be 95% certain of having enough money to pay the life annuities. (Note: the 95[th] percentile of a standard Normal is 1.645.)

Let $Y =$ total $PV = 10Y_1 + \ldots + 10Y_{25}$

where $Y_i = \ddot{a}_{\overline{K+1}|}$ for all i

$$EY_i = \ddot{a}_x = \frac{1-A_x}{d} = \frac{1-0.369131}{.06/1.06} = 11.1453523$$

$$\mathrm{Var}Y_i = \frac{{}^2A_x - A_x^2}{d^2} = \frac{0.1774113-(0.369131)^2}{(.06/1.06)^2}$$

$$= 12.8444973$$

Let $F =$ funds

$$P(F \geq Y) = P\left(\frac{Y-EY}{\sqrt{\mathrm{Var}Y}} \leq \frac{F - \overbrace{10(25)(11.1453523)}^{2768.33808}}{\underbrace{\sqrt{10^2(25)(12.8444973)}}_{179.196103}}\right)$$

$$\frac{F - 2768.33808}{179.196103} = 1.645$$

$\therefore F = 3{,}081.12$

Question No. 5:

You are given the following extracted from a mortality table:

x	q_x
40	.010
41	.015
42	.020
43	.025

Calculate $\ddot{a}_{40:\overline{3}|}$ if $i = 10\%$.

$$\ddot{a}_{40:\overline{3}|} = 1 + v\,p_{40} + v^2\,p_{40}\,p_{41}$$ by current payment technique

$$= 1 + \frac{1}{1.10}(.99) + \frac{1}{1.10^2}(.99)(.985)$$

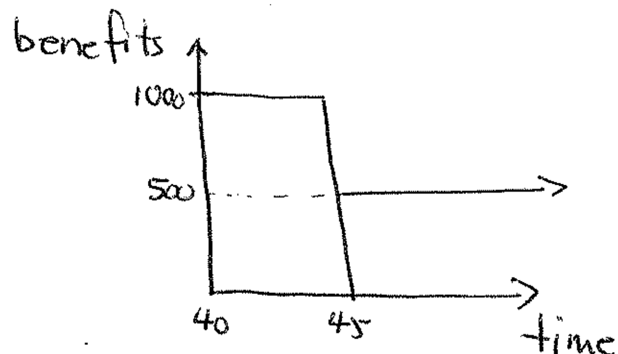
$$= \underline{\underline{2.7059}}$$

Question No. 6:

For a special type of whole life insurance issued to (40), you are given:

- death benefits are 1,000 for the first 5 years and 500 thereafter;
- death benefits are payable at the end of the year of death;
- mortality follows the *Illustrative Life table*; and
- $i = 6\%$.

Calculate the actuarial present value of the benefits for this policy.



APV(benefits)

$$= 1000 A_{40} - 500\ {}_5E_{40} A_{45}$$

$$= 1000\left(\frac{161.32}{1000}\right) - 500\left(\frac{735.29}{1000}\right)\left(\frac{201.20}{1000}\right)$$

$$= \underline{\underline{87.35}}$$

Question No. 7:

After calculating the value of $\ddot{a}_x$ at interest rate $i = 5\%$, a student discovers that the value of p_{x+1} is larger by 0.03 than the value used in the initial calculation.

You are given the following values used in the initial calculation:

$$q_x = 0.01, \quad q_{x+1} = 0.05, \quad \text{and} \quad \ddot{a}_{x+1} = 6.951.$$

Find the amount by which the value of $\ddot{a}_x$ is increased when the correct value of p_{x+1} is used.

Put a $*$ on values calculated based on the correct value of p_{x+1}

$\therefore$ By recursion, $\ddot{a}_x^* = 1 + vp_x + v^2 p_x \underbrace{p_{x+1}^*}_{(p_{x+1}+.03)} \underbrace{\ddot{a}_{x+2}}_{\text{not affected by } p_{x+1}}$

$$= \underbrace{1 + vp_x + v^2 p_x p_{x+1} \ddot{a}_{x+2}}_{\ddot{a}_x} + v^2 p_x (.03) \ddot{a}_{x+2}$$

$$\therefore \underbrace{\ddot{a}_x^* - \ddot{a}_x}_{\text{the required increase}} = v^2 p_x (.03) \ddot{a}_{x+2}$$

$$= .03 \frac{vp_x}{p_{x+1}} \underbrace{(vp_{x+1} \ddot{a}_{x+2})}_{\ddot{a}_{x+1} - 1}$$

$$= .03 \frac{1}{1.05} \frac{(.99)}{(.95)} (6.951 - 1)$$

$$= \underline{0.1772}$$

The life annuity is supposed to be higher than it was because one of the survival probabilities is <u>higher</u>.

Question No. 8:

Michel is currently age 40. His survival pattern follows DeMoivre's law with $\omega = 100$.

He purchases a three-year temporary life annuity that pays a benefit of 100 at the beginning of each year.

Compute the actuarial present value of his benefits if $i = 5\%$.

Let T_{40} be Michel's future lifetime. Since DeMoivre's, we have

$$T_{40} \sim \text{Uniform on } (0,60) \Rightarrow {}_kp_{40} = P(T_{40} > k) = 1 - k/60$$

Thus,

$$\text{APV(benefits)} = 100 * \sum_{k=0}^{2} v^k \, {}_kp_{40}$$

$$= 100 * \left(1 + v\,p_{40} + v^2 \, {}_2p_{40}\right)$$

$$= 100 * \left(1 + \frac{1}{1.05}\,\frac{59}{60} + \frac{1}{1.05^2}\,\frac{58}{60}\right)$$

$$= 100 * \underline{\underline{2.8133}} = \underline{281.33}$$

Question No. 9:

You are given:

- deaths are uniformly distributed over each year of age;
- $i = .06$;
- $q_{69} = 0.02$; and
- $\bar{A}_{70} = 0.53$.

Calculate $A_{69}^{(2)}$ and interpret this value.

By UDD, we have $A_{69}^{(2)} = \frac{i}{i^{(2)}} A_{69}$ where $i = .06$ and

$$i^{(2)} = 2\left[1.06^{1/2} - 1\right] = .05912603$$

also, $\bar{A}_{70} = \frac{i}{\delta} A_{70} \Rightarrow A_{70} = \frac{\delta}{i} \bar{A}_{70}$

$$= \frac{\log 1.06}{.06}(.53) = 0.51470869$$

∴ By recursion, $A_{69} = v q_{69} + v p_{69} A_{70}$

$$= \frac{1}{1.06}(.02) + \frac{1}{1.06}(.98)(.51470869)$$

$$= 0.49473067$$

$$\therefore A_{69}^{(2)} = \frac{.06}{.05912603}(.49473067) = \underline{\underline{.5020}}$$

this gives the APV of a whole life insurance of \$1 issued to (69) with death benefit paid at the end of the semiannual in the year of death.

Question No. 10:

You are given:

- $\ddot{a}_{60:\overline{10|}} = 6.4745$;
- $A^{\,1}_{60:\overline{10|}} = 0.0786$; and
- $d = 0.0909$.

Calculate the actuarial present value of a 10-year pure endowment issued to (60).

Want ${}_{10}E_{60} = ?$

We know $\underbrace{A_{60:\overline{10|}}}_{A^{1}_{60:\overline{10|}} + {}_{10}E_{60}} = 1 - d\,\ddot{a}_{60:\overline{10|}}$

$$\Rightarrow \quad {}_{10}E_{60} = 1 - d\,\ddot{a}_{60:\overline{10|}} - A^{1}_{60:\overline{10|}}$$

$$= 1 - .0909(6.4745) - 0.0786$$

$$= \underline{\underline{0.3329}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK