MATH 3630
Actuarial Mathematics I
Class Test 2
Wednesday, 16 November 2011
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: SOLUTIONS

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

You are given:

- $\ddot{a}_{x:\overline{n}|} = 15.884$
- ${}_nE_x = 0.336$
- $i = 4\%$

Calculate $A^{1}_{x:\overline{n}|}$ and interpret this value.

$$A_{x:\overline{n}|} = 1 - d\,\ddot{a}_{x:\overline{n}|}$$

$$= 1-(1-(1.04)^{-1})(15.884)$$

$$= 0.3890769$$

$$A^{1}_{x:\overline{n}|} = A_{x:\overline{n}|} - {}_nE_x$$

$$= 0.3890769 - 0.336$$

$$= 0.05307692$$

This value gives the actuarial present value of an n-year term insurance to (x), with benefit of \$1 payable at the end of the year of death provided death occurs before n years.

Question No. 2:

Suppose you are given the following extract from a select-and-ultimate mortality table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	x+3
55	882	877	871	864	58
56	875	870	863	856	59
57	868	863	856	849	60
58	861	855	848	840	61
59	854	847	840	832	62
60	846	839	832	823	63

Calculate $\ddot{a}_{[57]:\overline{3}|}$ if $i = 5\%$.

use current payment technique

$$\ddot{a}_{[57]:\overline{3}|} = \sum_{k=0}^{2} v^k \, {}_kp_{[57]}$$

$$= 1 + v\, p_{[57]} + v^2\, p_{[57]}\, p_{[57]+1}$$

$$= 1 + v \frac{\ell_{[57]+1}}{\ell_{[57]}} + v^2 \frac{\ell_{[57]+2}}{\ell_{[57]}}$$

$$= 1 + (1.05)^{-1} \frac{863}{868} + (1.05)^{-2} \frac{856}{868}$$

$$= \underline{\underline{2.841385}}$$

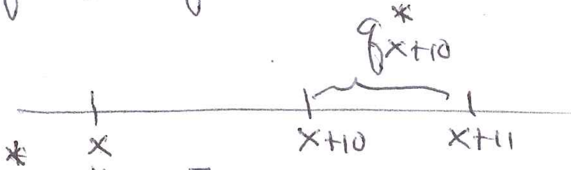
Question No. 3:

You are given:

- $q_{x+10} = 0.004$
- $\ddot{a}_{x+10} = 13.14$
- ${}_{10}E_x = 0.49$
- $i = 7\%$

If q_{x+10} is increased by a constant 0.001 and everything else remains, how much will $\ddot{a}_x$ decrease by?

Let $\ddot{a}_x^*$ be the new $\ddot{a}_x$ when $q_{x+10}^* = q_{x+10} + .001$



Using recursion,

$$\ddot{a}_x^* = \ddot{a}_{x:\overline{10}|} + {}_{10}E_x \left[1 + v \underbrace{p^*_{x+10}}_{1-q^*_{x+10} = 1 - q_{x+10} - .001 = p_{x+10} - .001} \ddot{a}_{x+11} \right]$$

$$= \underbrace{\ddot{a}_{x:\overline{10}|} + {}_{10}E_x \left[1 + v p_{x+10} \ddot{a}_{x+11} \right]}_{\ddot{a}_x} - .001\, v\, {}_{10}E_x\, \ddot{a}_{x+11}$$

Decrease is then

$$\ddot{a}_x - \ddot{a}_x^* = .001\, v\, {}_{10}E_x\, \ddot{a}_{x+11}$$

$$= \underbrace{.001 (1.07)^{-1} (0.49)(13.04197)}$$

$$= \underline{\underline{.005972491}}$$

$$\ddot{a}_{x+10} = 1 + v p_{x+10} \ddot{a}_{x+11}$$

$$\ddot{a}_{x+11} = \frac{\ddot{a}_{x+10} - 1}{v p_{x+10}} = \frac{13.14 - 1}{(1.07)^{-1}(1 - .004)} = 13.04197$$

Question No. 4:

Ms. Barbee Cue purchases a state-of-the-art television set for the price of \$10,000. For peace of mind, she buys with it a three-year warranty insurance which will replace the product at the end of the year it fails, if it fails within the next three years.

Assume the cost of the same product will not change the next three years and that $i = 5\%$.

Denote by K the end of the year of failure of the product. You are given:

k	$\Pr[K = k]$
1	0.05
2	0.10
3	0.20

Calculate the standard deviation of the present value of Ms. Cue's warranty benefits.

K	Pr[K=k]	y	y·Pr[K=k]	y²·Pr[K=k]
1	.05	v = .9523810	.04761905	.04535147
2	.10	v² = .9070295	.09070295	.08227025
3	.20	v³ = .8638376	.17276752	.14924308
≥4	.65	∅	∅	∅
		sum	0.3110895	0.2768648

Let Y = p.v. of \$1 of warranty benefit

$E[Y] = 0.3110895$

$Var[Y] = E[Y^2] - E[Y]^2 = 0.2768648 - (0.3110895)^2$

$= 0.1800881$

The standard deviation of the p.v. of Ms. Cue's warranty benefits is

$10000 * \sqrt{Var[Y]} = 10000 * \sqrt{0.1800881}$

$= \underline{\underline{4,243.679}}$

Question No. 5:

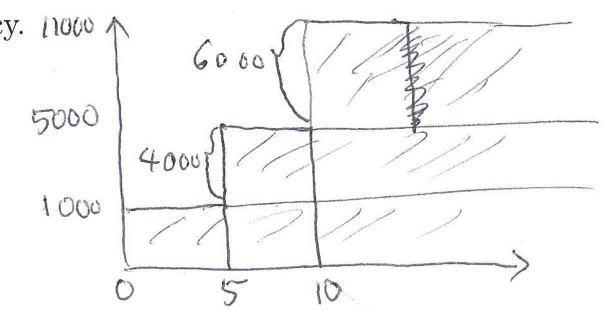
Get-a-Life Insurance Company issues a special whole life insurance policy to Mr. Ow Sum, currently aged 50, which will pay him at the end of the year of his death the following benefits:

- \$1,000 if death occurs within the first 5 years,
- \$5,000 if death occurs within the subsequent 5 years, and
- \$11,000 if death occurs thereafter.

You are given following table:

x	$1000A_x$	$1000\,{}_5E_x$
50	241.29	764.15
55	287.88	757.17
60	341.96	746.05

Calculate the actuarial present value of Mr. Sum's policy.



$$APV(\text{policy}) = 1000A_{50} + 4000\,{}_5E_{50}\,A_{55} + 6000\,{}_5E_{50}\,{}_5E_{55}\,A_{60}$$

$$= 241.29 + 4(764.15)(.28788) + 6(764.15)(.75717)(.34196)$$

$$= \underline{\underline{2,308.355}}$$

Question No. 6:

Mr. Jack Pot, age x, just won the sweepstakes lottery for which he has the option of receiving his winnings in the form of either:

- a payment of \$2,000 at the beginning of each month that he is alive;

  or

- a guaranteed payment of \$20,000 at the beginning of each year for 10 years, plus an additional payment of b at the beginning of each year that he is alive in subsequent years.

The two payment options have equal actuarial present values.

You are given: $\ddot{a}_{\overline{10}|} = 8.1$, ${}_{10}E_x = 0.6$, $\ddot{a}_x^{(12)} = 18.0$, and $\ddot{a}_{x+10} = 17.4$.

Calculate b.

First option: $APV = 2{,}000(12)\,\ddot{a}_x^{(12)} = 24000(18) = 432{,}000$

Second option: $APV = 20{,}000\,\ddot{a}_{\overline{10}|} + b \cdot {}_{10}E_x\,\ddot{a}_{x+10}$

$= 20{,}000\,(8.1) + b\,(0.6)(17.4)$

Equating the two APVs and solving for b, we get

$$b = \underbrace{\frac{432{,}000 - 20{,}000(8.1)}{0.6\,(17.4)}}$$

$$= \underline{\underline{25{,}862.07}}$$

Question No. 7:

For a cohort of individuals all age x consisting of 65% males (m) and 35% females (f), you are given:

k	q_{x+k}^{m}	q_{x+k}^{f}
0	0.04	0.01
1	0.08	0.04
2	0.12	0.07

In addition:

- Mortality follows uniform distribution of deaths for integral ages.
- $i = 3\%$

Calculate $\ddot{a}_{x:\overline{2}|}^{(12)}$ for a randomly chosen individual from this cohort.

Note: For $i = 3\%$, you may use $i^{(12)} = 0.0296$ and $d^{(12)} = 0.0295$.

males: $${}^{m}A_{x:\overline{2}|}^{(12)} = \frac{i}{i^{(12)}}\,{}^{m}A_{x:\overline{2}|}^{\;\;1} + {}^{m}{}_2E_x = \underbrace{\frac{.03}{.0296}\left[(1.03)^{-1}(.04) + (1.03)^{-2}(.96)(.08)\right] + (1.03)^{-2}(.96)(.92)}_{.9452301}$$

females: $${}^{f}A_{x:\overline{2}|}^{(12)} = \underbrace{\frac{.03}{.0296}\left[(1.03)^{-1}(.01) + (1.03)^{-2}(.99)(.04)\right] + (1.03)^{-2}(.99)(.96)}$$

$$= .9435143$$

$$A_{x:\overline{2}|}^{(12)} = .9452301(.65) + .9435143(.35) = 0.9446296$$

$$\ddot{a}_{x:\overline{2}|}^{(12)} = \frac{1 - A_{x:\overline{2}|}^{(12)}}{d^{(12)}} = \frac{1 - 0.9446296}{.0295} = \underline{\underline{1.876964}}$$

Question No. 8:

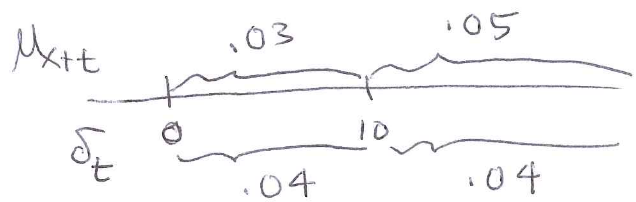
For a whole life insurance of \$1 on (x) with benefits payable at the moment of death, you are given:

$$\mu_{x+t} = \begin{cases} 0.03, & 0 < t \leq 10 \\ 0.05, & t > 10 \end{cases}$$

and

$$\delta_t = 0.04, \quad \text{for} \quad t > 0.$$

Calculate the actuarial present value for this insurance.



$$\text{APV(insurance)} = \frac{.03}{.07}\left[1 - e^{-.07(10)}\right] + e^{-.07(10)}\left(\frac{.05}{.09}\right)$$

$$= \underline{\underline{0.4916299}}$$

Question No. 9:

For a whole life annuity that pays \$1 at the beginning of each year that (65) is alive, you are given:

- Y is the present value random variable for this annuity.
- Mortality follows the `Illustrative Life Table` with $i = 6\%$.

Calculate Var[Y].

$$Y = \ddot{a}_{\overline{K+1}|} = \frac{1-v^{K+1}}{d}$$

$$Var[Y] = \frac{1}{d^2} Var[v^{K+1}] = \frac{1}{d^2}\left[{}^2A_{65} - (A_{65})^2\right]$$

$$= \frac{1}{(1-(1.06)^{-1})^2}\left[.23603 - (.43980)^2\right]$$

$$= \underline{\underline{13.29779}}$$

Question No. 10:

For a whole life insurance of $1 with benefits payable at the moment of death of (x), you are given:

- Z is the present value random variable for this insurance.
- T_x is the future lifetime random variable for (x).
- T_x has an Exponential distribution with constant force of mortality of 0.01.
- $\delta = 5\%$

Calculate $\Pr[Z \leq 0.50]$.

$$\Pr[Z \leq 0.50] = \Pr[e^{-\delta T} \leq 0.50]$$
$$= \Pr[-\delta T \leq \log(0.50)]$$
$$= \Pr\left[T \geq \frac{\log(0.50)}{-.05}\right]$$
$$= \Pr\left[T \geq \underbrace{\log(0.50)^{-1/.05}}\right]$$
$$\log\left(\tfrac{1}{2}\right)^{-20} = \log 2^{20}$$

Thus,

$$\Pr[Z \leq 0.50] = e^{-(\log 2^{20})(.01)} = 2^{-.01(20)}$$
$$= \underline{\underline{0.8705506}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK