Michigan State University

STT 455 - Actuarial Models I

Class Test 2

Monday, 11 November 2013

Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTIONS Section No.: ________________

- There are five (5) multiple choice (MC) and one (1) written-answer questions here and you are to answer all questions asked. Points assigned are clearly indicated on each question.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.
- The Illustrative Life Table (ILT) is attached in the last two pages of this paper.

MC Question No. 1: (10 points)

You are given:

- $q_{x+k} = 0.10(1+k)$, for $k = 0, 1, 2$
- $i = 0.05$

$\Rightarrow$ $q_x = 0.10$

$q_{x+1} = 0.20$

$q_{x+2} = 0.30$

Calculate the actuarial present value of a three-year endowment insurance of \$1 issued to (x).

(a) 0.46

(b) 0.60

(c) 0.74

(d) 0.88

(e) 1.02

$$A_{x:\overline{3}|} = v q_x + v^2 p_x q_{x+1} + v^3 p_x p_{x+1} q_{x+2} + v^3 p_x p_{x+1} p_{x+2}$$

$$= v q_x + v^2 p_x q_{x+1} + v^3 p_x p_{x+1} (\underbrace{q_{x+2} + p_{x+2}}_{1})$$

$$= \frac{1}{1.05} 0.10 + \frac{1}{1.05^2} 0.9(0.2) + \frac{1}{1.05^3} 0.9(0.8)$$

$$= \underline{\underline{0.8804665}}$$

MC Question No. 2: (10 points)

You are given:

- $A_{45} = 0.15$
- $p_{45} = 0.99$
- $i = 0.04$

Calculate $\ddot{a}_{46}$.

(a) 21.2

(b) 22.2

(c) 23.2

(d) 24.2

(e) 25.2

$$\ddot{a}_{45} = \frac{1 - A_{45}}{d} = \frac{1 - .15}{.04/1.04} = 22.1$$

Using recursion, we have

$$\ddot{a}_{45} = 1 + v p_{45} \ddot{a}_{46}$$

Solving for $\ddot{a}_{46}$, we get

$$\ddot{a}_{46} = \frac{\ddot{a}_{45} - 1}{v p_{45}}$$

$$= \frac{22.1 - 1}{\frac{1}{1.04}(.99)}$$

$$= 22.16566 \approx \underline{\underline{22.2}}$$

MC Question No. 3: (10 points)

You are given:

- T_x, the future lifetime of (x), has an Exponential distribution with $\mu = 0.15$.
- $\delta = 0.05$
- Z is the present value random variable for a whole life insurance of \$10 payable at the moment of death of (x).

Calculate the probability that Z will exceed \$5.

(a) 0.125

(b) 0.375

(c) 0.555

(d) 0.875

(e) 0.925

$$Z = 10e^{-.05T_x}$$

$$\Pr[Z>5] = \Pr[10e^{-.05T_x} > 5]$$

$$= \Pr[\underbrace{e^{-.05T_x} > 0.5}]$$

$$\Leftrightarrow -.05T_x > \log 2^{-1}$$

$$\Leftrightarrow T_x < \frac{-1}{.05}\log 2^{-1} = \log 2^{+1/.05}$$

Exponential $\mu = 0.15$

$$\Rightarrow \Pr[T_x < \log 2^{1/.05}] = 1 - e^{-0.15\log 2^{1/.05}}$$

$$= 1 - e^{\log 2^{-.15/.05}} = 1 - 2^{-3} = 1 - 1/8$$

$$= 7/8$$

$$= \underline{\underline{.875}}$$

MC Question No. 4: (10 points)

For a special whole life insurance on (50), you are given:

- The death benefit, payable at the end of the year of death, is 100 if death occurs within the first 10 years and 250 if death occurs thereafter.
- Mortality follows the *Illustrative Life Table*.
- $i = 0.06$

Calculate the actuarial present value for this life insurance.

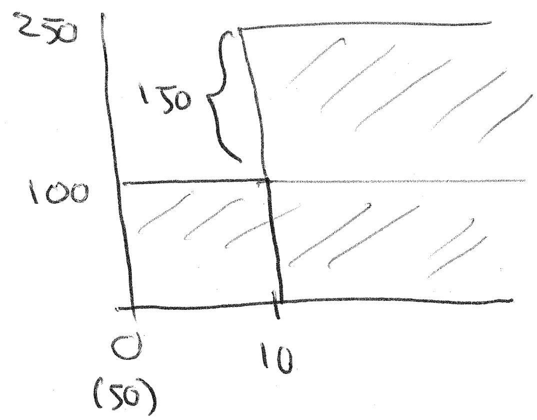
(a) 34.3

(b) 43.8

(c) 53.2

(d) 64.6

(e) 72.0



$$APV = 100\, A_{50} + 150\ {}_{10}E_{50}\, A_{60}$$

$$= 100(.24905) + 150(.51081)(.36913)$$

$$= 53.18829 \approx \underline{\underline{53.2}}$$

Question No. 5: (10 points)

You are given:

- $i = 0.05$
- $A_{50} = 0.21$
- $A_{51} = 0.20$
- ${}^2A_{51} = 0.06$
- Z is the present value random variable for a whole life insurance of \$1 issued to (50).

Calculate the variance of Z.

(a) 0.01

(b) 0.02

(c) 0.03

(d) 0.04

(e) 0.05

$$A_{50} = v\overset{1-p_{50}}{q_{50}} + v p_{50} A_{51}$$

$$= v - v p_{50}(1 - A_{51})$$

$$\Rightarrow p_{50} = \frac{(1+i)A_{50} - 1}{A_{51} - 1} = 0.974375$$

$$Var[Z] = {}^2A_{50} - (A_{50})^2$$

$${}^2A_{50} = v^2 q_{50} + v^2 p_{50}\,{}^2A_{51}$$

$$= \underbrace{\frac{1}{1.05^2}\left[(1-.974375) + .974375(.06)\right]}_{.07626984}$$

$$= .07626984 - (.21)^2$$

$$= .03216984 \simeq \underline{\underline{0.03}}$$

Note: This is present value for a whole life that pays end of year of death. I think this is clearly implied by the problem, knowing that all givens are end of year, and no assumption to convert something other than end of year of death.

There are five (5) parts to the written-answer portion of this test and you are to answer all parts. Please provide as much details of your calculations as possible to get your partial points for any incorrect answers.

You are given:

- $i = 4\%$ and the following table:

d_x

x	ℓ_x	d_x	μ_x
55	8755	97	0.0104
56	8658	107	0.0117
57	8551	119	0.0132
58	8432		0.0149

(i) (10 points) Assuming Uniform Distribution of Deaths (UDD) between integral ages, calculate $\bar{A}^{1}_{55:\overline{3}|}$ and interpret this value.

$$\bar{A}^{1}_{55:\overline{3}|} = \frac{i}{\delta} A^{1}_{55:\overline{3}|}$$

$$= \frac{i}{\delta}\left[v\, q_{55} + v^2 p_{55}\, q_{56} + v^3 p_{55}\, p_{56}\, q_{57} \right]$$

$$= \underbrace{\frac{.04}{\log(1.04)}\left[\frac{1}{1.04}\frac{97}{8755} + \frac{1}{1.04^2}\frac{\cancel{8658}}{8755}\frac{107}{\cancel{8658}} + \frac{1}{1.04^3}\frac{119}{8755} \right]}$$

$.03471252$

This gives the actuarial present value of a 3-year term insurance to (55) of \$1 payable at the end of year of death.

(ii) (10 points) Calculate the exact value of $\ddot{a}_{55:\overline{3}|}$.

$$\ddot{a}_{55:\overline{3}|} = 1 + v\,p_{55} + v^2\,p_{55}\,p_{56}$$

$$= 1 + \frac{1}{1.04}\cdot\frac{8658}{8755} + \frac{1}{1.04^2}\,\frac{\cancel{8658}}{8755}\,\frac{8551}{\cancel{8658}}$$

$$= \underline{\underline{2.853898}}$$

alternatively, use $A^{\,1}_{55:\overline{3}|} = .03403624$ which can be easily obtained from (i)

Then, $A_{55:\overline{3}|} = A^{\,1}_{55:\overline{3}|} + v^3\,{}_3p_{55}$

$$= A^{\,1}_{55:\overline{3}|} + \frac{1}{(1+i)^3}\,\frac{l_{58}}{l_{55}}$$

$$= .03403624 + \frac{1}{1.04^3}\,\frac{8432}{8755} = 0.8902347$$

Using relationship between A & $\ddot{a}$, we get

$$\ddot{a}_{55:\overline{3}|} = \frac{1 - A_{55:\overline{3}|}}{d} = \frac{1 - 0.8902347}{.04/1.04} = \underline{\underline{2.853898}}$$

same answer!

(iii) (10 points) Assuming Uniform Distribution of Deaths (UDD) between integral ages, calculate $\ddot{a}^{(12)}_{55:\overline{3|}}$.

Hint: You may use the result that when $i = 0.04$, $\alpha(12) = 1.000127$ and $\beta(12) = 0.464889$.

$$\ddot{a}^{(12)}_{55:\overline{3|}} = \alpha(12)\,\ddot{a}_{55:\overline{3|}} - (1 - {}_3E_{55})\,\beta(12)$$

$$v^3 \frac{l_{58}}{l_{55}} = \frac{1}{1.04^3}\,\frac{8432}{8755} = .8561984$$

$$= 1.000127\,(2.853898) - \left(1 - \frac{1}{1.04^3}\,\frac{8432}{8755}\right)(.464889)$$

$$= \underline{\underline{2.787409}}$$

(iv) (15 points) Using Woolhouse approximation formula based on the first 3 terms (W3), calculate $\ddot{a}^{(12)}_{55:\overline{3}|}$.

Woolhouse formula with 3 terms

.8561984 see previous page

$$\ddot{a}^{(12)}_{55:\overline{3}|} = \underbrace{\ddot{a}_{55:\overline{3}|}}_{2.853898} - \frac{12-1}{2(12)}\left(1 - {}_3E_{55}\right)$$

$$- \frac{12^2-1}{12(12^2)}\left[\underbrace{\delta}_{\log(1.04)} + \underbrace{\mu_{55}}_{.0104} - {}_3E_{55}\left(\delta + \underbrace{\mu_{58}}_{.0149}\right)\right]$$

$\Rightarrow$ plug the values, we get

$$\ddot{a}^{(12)}_{55:\overline{3}|} = \underline{\underline{2.788261}}$$ using Woolhouse with 3 terms

(v) (5 points) Explain why the Woolhouse formula may be a better approximation than that based on the UDD assumption.

UDD assumes deaths are always spread evenly over each year

We know such is not generally true, especially in later ages

Woolhouse (W3) formula contains a μ that better reflects the increasing likelihood of dying in a year, especially in later ages

This may explain why W3 approximation performs better than UDD approximation!

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
0	10,000,000	20.42	16.8010	49.00	25.92	728.54	541.95	299.89	0
5	9,749,503	0.98	17.0379	35.59	8.45	743.89	553.48	305.90	5
10	9,705,588	0.85	16.9119	42.72	9.37	744.04	553.34	305.24	10
15	9,663,731	0.91	16.7384	52.55	11.33	743.71	552.69	303.96	15
20	9,617,802	1.03	16.5133	65.28	14.30	743.16	551.64	301.93	20
21	9,607,896	1.06	16.4611	68.24	15.06	743.01	551.36	301.40	21
22	9,597,695	1.10	16.4061	71.35	15.87	742.86	551.06	300.82	22
23	9,587,169	1.13	16.3484	74.62	16.76	742.68	550.73	300.19	23
24	9,576,288	1.18	16.2878	78.05	17.71	742.49	550.36	299.49	24
25	9,565,017	1.22	16.2242	81.65	18.75	742.29	549.97	298.73	25
26	9,553,319	1.27	16.1574	85.43	19.87	742.06	549.53	297.90	26
27	9,541,153	1.33	16.0873	89.40	21.07	741.81	549.05	297.00	27
28	9,528,475	1.39	16.0139	93.56	22.38	741.54	548.53	296.01	28
29	9,515,235	1.46	15.9368	97.92	23.79	741.24	547.96	294.92	29
30	9,501,381	1.53	15.8561	102.48	25.31	740.91	547.33	293.74	30
31	9,486,854	1.61	15.7716	107.27	26.95	740.55	546.65	292.45	31
32	9,471,591	1.70	15.6831	112.28	28.72	740.16	545.90	291.04	32
33	9,455,522	1.79	15.5906	117.51	30.63	739.72	545.07	289.50	33
34	9,438,571	1.90	15.4938	122.99	32.68	739.25	544.17	287.82	34
35	9,420,657	2.01	15.3926	128.72	34.88	738.73	543.18	286.00	35
36	9,401,688	2.14	15.2870	134.70	37.26	738.16	542.11	284.00	36
37	9,381,566	2.28	15.1767	140.94	39.81	737.54	540.92	281.84	37
38	9,360,184	2.43	15.0616	147.46	42.55	736.86	539.63	279.48	38
39	9,337,427	2.60	14.9416	154.25	45.48	736.11	538.22	276.92	39
40	9,313,166	2.78	14.8166	161.32	48.63	735.29	536.67	274.14	40
41	9,287,264	2.98	14.6864	168.69	52.01	734.40	534.99	271.12	41
42	9,259,571	3.20	14.5510	176.36	55.62	733.42	533.14	267.85	42
43	9,229,925	3.44	14.4102	184.33	59.48	732.34	531.12	264.31	43
44	9,198,149	3.71	14.2639	192.61	63.61	731.17	528.92	260.48	44
45	9,164,051	4.00	14.1121	201.20	68.02	729.88	526.52	256.34	45
46	9,127,426	4.31	13.9546	210.12	72.72	728.47	523.89	251.88	46
47	9,088,049	4.66	13.7914	219.36	77.73	726.93	521.03	247.08	47
48	9,045,679	5.04	13.6224	228.92	83.06	725.24	517.91	241.93	48
49	9,000,057	5.46	13.4475	238.82	88.73	723.39	514.51	236.39	49
50	8,950,901	5.92	13.2668	249.05	94.76	721.37	510.81	230.47	50
51	8,897,913	6.42	13.0803	259.61	101.15	719.17	506.78	224.15	51
52	8,840,770	6.97	12.8879	270.50	107.92	716.76	502.40	217.42	52
53	8,779,128	7.58	12.6896	281.72	115.09	714.12	497.64	210.27	53
54	8,712,621	8.24	12.4856	293.27	122.67	711.24	492.47	202.70	54
55	8,640,861	8.96	12.2758	305.14	130.67	708.10	486.86	194.72	55
56	8,563,435	9.75	12.0604	317.33	139.11	704.67	480.79	186.32	56
57	8,479,908	10.62	11.8395	329.84	147.99	700.93	474.22	177.53	57
58	8,389,826	11.58	11.6133	342.65	157.33	696.85	467.12	168.37	58
59	8,292,713	12.62	11.3818	355.75	167.13	692.41	459.46	158.87	59
60	8,188,074	13.76	11.1454	369.13	177.41	687.56	451.20	149.06	60
61	8,075,403	15.01	10.9041	382.79	188.17	682.29	442.31	139.00	61
62	7,954,179	16.38	10.6584	396.70	199.41	676.56	432.77	128.75	62
63	7,823,879	17.88	10.4084	410.85	211.13	670.33	422.54	118.38	63
64	7,683,979	19.52	10.1544	425.22	223.34	663.56	411.61	107.97	64
65	7,533,964	21.32	9.8969	439.80	236.03	656.23	399.94	97.60	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90
91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110