MATH 3630
Actuarial Mathematics I
Final Examination - sec 001
Monday, 10 December 2012
Time Allowed: 2 hours (6:00 - 8:00 pm)
Room: MSB 411
Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: ________________________ **Student ID:** __________________

- **There are twelve (12) written-answer questions here and you are to answer all twelve. Each question is worth 10 points. Your final mark will be divided by 120 to convert to a unit of 100%.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**
- **Good luck.**
- **Have a Happy and Healthy Christmas and New Year!**

Question	Worth	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	
%	÷ 120	

Question No. 1:

You are given:

- Mortality follows Gompertz' law, $\mu_x = Bc^x$, with $B = 0.0001$ and $c = 1.10$.
- $\delta = 5\%$
- $\ddot{a}_{50} = 12.09$
- $\ddot{a}_{65} = 7.55$

Evaluate $\ddot{a}^{(4)}_{50:\overline{15}|}$ based on Woolhouse's approximation with three terms and interpret this value.

Question No. 2:

For a special whole life insurance policy issued to (45), you are given:

- Death benefits are payable at the end of the year of death while level premiums are payable once a year.
- The amount of benefit is $200,000 if death occurs within the first 20 years and reduces to $100,000 for death thereafter.
- Mortality follows the `Illustrative Life Table`.
- $i = 6\%$

Calculate the annual benefit premium based on the actuarial equivalence principle.

Question No. 3:

For a fully discrete whole life insurance policy of \$1,000 on (x), you are given:

- The annual benefit premium is
$$P^* = P + 2,$$
where P is the annual benefit premium determined according to the equivalence principle.
- $i = 0.10$
- $A_x = 0.5$
- ${}^2A_x = 0.3$

Calculate the variance of the loss-at-issue.

Question No. 4:

For a whole life annuity on (x) with benefits continuously paid at the rate of \$12,000 per year, you are given:

$$\delta_t = \begin{cases} 0.01, & 0 < t \leq 10 \\ 0.04, & t > 10 \end{cases}$$

and

$$\mu_{x+t} = \begin{cases} 0.005, & 0 < t \leq 5 \\ 0.010, & t > 5 \end{cases}$$

Calculate the actuarial present value for this annuity.

Question No. 5:

For a special two-year term life insurance policy issued to (40), you are given:

- Benefit of $10,000 is payable at the moment of death.
- Premiums are payable semi-annual.
- The following is an extract from the mortality table:

x	40	41	42
ℓ_x	100	98	95

- Mortality follows a constant force assumption over each year of age.
- $i^{(2)} = 0.06$ [Note: The corresponding force of interest is $\delta = 0.05912$.]
- Premiums are based on the actuarial equivalence principle.

Calculate the amount of the semi-annual premium for this policy.

THIS PAGE FOR EXTRA SPACE TO SOLVE QUESTION 5

Question No. 6:

For a fully discrete whole life insurance of $1 issued to (x), you are given:

- L_0 is the net loss-at-issue random variable with the premium determined according to the actuarial equivalence principle.
- L_0^* is the net loss-at-issue random variable with the premium determined such that $\mathrm{E}\left[L_0^*\right] = -0.5$.
- $\mathrm{Var}\left[L_0\right] = 0.75$.

Calculate $\mathrm{Var}\left[L_0^*\right]$.

Question No. 7:

Get-a-Life Insurance Company issues a special 10-year endowment insurance policy to a person now age 45. You are given:

- Premiums are payable once a year, where the first year premium is P and the subsequent premiums are each year equal to half of the first year.
- If death occurs within the first 10 years, the amount of benefit is $1,000,000 plus the difference between the first and second year premiums.
- An endowment equal to the first year premium is paid at the end of 10 years.
- Mortality follows the `Standard Ultimate Survival Model` with $i = 5\%$.

Calculate P.

Question No. 8:

For a cohort of individuals all age x consisting of non-smokers (ns) and smokers (sm), you are given:

- Mortality is based on the following:

k	q_{x+k}^{ns}	q_{x+k}^{sm}
0	0.01	0.08
1	0.03	0.12

- $i = 5\%$
- $A_{x:\overline{2}|}^{\,1} = 0.0616$ for a randomly chosen individual from this cohort

Determine the proportion of non-smokers and smokers in this cohort at age x.

Question No. 9:

For a special 3-year temporary life annuity on (65), you are given:

- The annuity payments are $20, $30, and $50, respectively, payable at the beginning of each year while (65) is alive. No further payments made after 3 years.

- Mortality is based on the following extract from a life table:

x	65	66	67	68
ℓ_x	9500	9400	9200	8900

- $i = 4\%$

- Y is the present value random variable for this annuity.

Calculate $\text{Var}[Y]$.

Question No. 10:

Christian is currently age 45 who purchases a special deferred whole life annuity-due policy which will pay him the following benefits:

- guaranteed annual payments of B for 5 years, with the first payment to start when he reaches age 65; and
- annual payments of $100,000, if alive, thereafter.

You are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 6\%$
- The actuarial present value of Christian's life annuity benefits is $225,000.

Calculate B.

Question No. 11:

For a fully discrete whole life insurance of $1 on (50), you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 6\%$
- The annual benefit premium is equal to 0.02.

Calculate the probability of a positive loss-at-issue.

Question No. 12:

Suppose you are given:

- $e_x = 30.0$
- $e_{x+1} = 30.6$
- $e_{x+2} = 32.9$

Calculate the probability that (x) will survive the next year but dies the following year.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK