STT 455 Review Session for Class Test 2
November 13, 2014

1. For a special whole life insurance on (45), you are given:

   - Death benefit is payable at the end of the year of death.
   - Death benefit is \$10,000 during the first 10 years, and \$20,000 thereafter.
   - Mortality follows the `Illustrative Life Table`.
   - $i = 6\%$

   Calculate the actuarial present value of this insurance.

2. For a cohort of individuals all age $x$ consisting of 80% non-smokers (ns) and 20% smokers (sm), you are given:

   | $k$ | $q_{x+k}^{\text{ns}}$ | $q_{x+k}^{\text{sm}}$ |
   |---|---|---|
   | 0 | 0.02 | 0.08 |
   | 1 | 0.04 | 0.15 |
   | 2 | 0.06 | 0.20 |

   Calculate $A^{\,1}_{x:\overline{2}|}$ for a randomly chosen individual from this cohort. You are given: $i = 3\%$.

3. For a whole life insurance of \$100 issued to (65), you are given:

   - Death benefits are payable at the end of the year of death.
   - Mortality follows the `Illustrative Life Table` with the exception of the first year where you are given that $q_{65} = 0.03$.
   - The annual effective interest rate is 2% in the first year, 3% in the second year, and 6% each year thereafter.

   Calculate the actuarial present value of the death benefits.

4. You are given:

   - $\ddot{a}_{60:\overline{10}|} = 6.50$;
   - $A^{\,1}_{60:\overline{10}|} = 0.08$; and
   - $v = 0.92$.

Calculate the actuarial present value of a 10-year pure endowment issued to (60). Calculate the variance of the present value of the benefits for this same pure endowment.

5. You are given:

   - Deaths are uniformly distributed over each year of age.
   - $q_x = 0.05$
   - $q_{x+1} = 0.08$
   - $i = 5\%$

   (a) Evaluate $\bar{A}^{\,1}_{x:\overline{2}|}$.

   (b) Evaluate ${}^2\bar{A}^{\,1}_{x:\overline{2}|}$.

   (c) Explain verbally the benefits provided by $\bar{A}^{\,1}_{x:\overline{2}|}$.

6. You are given:

   - $Z$ is the present value random variable for a whole life insurance of 1 payable at the moment of death of (50).
   - Mortality follows de Moivre's law.
   - $\delta = 5\%$
   - The probability that $Z$ exceeds 0.0734 is 0.95.

   Calculate $\omega$ in the de Moivre's law.

7. Suppose you are given the following extract from a select-and-ultimate mortality table:

| $[x]$ | $\ell_{[x]}$ | $\ell_{[x]+1}$ | $\ell_{[x]+2}$ | $\ell_{x+3}$ | x+3 |
|---|---|---|---|---|---|
| 55 | 882 | 877 | 871 | 864 | 58 |
| 56 | 875 | 870 | 863 | 856 | 59 |
| 57 | 868 | 863 | 856 | 849 | 60 |
| 58 | 861 | 855 | 848 | 840 | 61 |
| 59 | 854 | 847 | 840 | 832 | 62 |
| 60 | 846 | 839 | 832 | 823 | 63 |

   Calculate $\ddot{a}_{[57]:\overline{3}|}$ if $i = 5\%$.

8. For a whole life annuity-due issued to (50), you are given:

   - The annual benefit is 100.
   - Mortality follows the *Illustrative Life Table*.
   - $i = 6\%$

Calculate the variance of the present value of the benefits for this annuity.

9. (See also Exercise 5.7) You are given:

   - $\ddot{a}_x = 11.2$
   - ${}_{15|}\ddot{a}_x = 4.5$
   - $A^{1}_{x:\overline{15|}} = 0.212$
   - ${}_{15}E_x = 0.255$

   Calculate $i$.

10. For a special 10-year term insurance on (35), you are given:

    - Death benefits are payable at the end of the year of death.
    - The death benefit is 100 in years 1-5 and increases to 200 in years 5-10.
    - Mortality follows the *Illustrative Life Table*.
    - $i = 6\%$
    - $Z$ denotes the present value, at age 35, of these death benefits.

    (a) Write an expression for $Z$ in terms of the curtate future lifetime of (35), say $K$.

    (b) Calculate $\Pr[Z = 0]$.

    (c) Calculate $\Pr[Z > 85]$.

    (d) Calculate $\mathrm{E}[Z]$.