Exercise 3.10

For $0 \leq t \leq 2$, we have

$$\begin{aligned}
{}_tp_{[x]} &= \exp\left\{-\int_0^t \mu_{[x]+s} ds\right\} \\
&= \exp\left\{-\int_0^t 0.9^{2-s}\mu_{x+s} ds\right\} \\
&= \exp\left\{-\int_0^t 0.9^{2-s}\left(A + Bc^{x+s}\right) ds\right\} \\
&= \exp\left\{-0.9^2\int_0^t \left(0.9^{-s}A + B0.9^{-s}c^{x+s}\right) ds\right\} \\
&= \exp\left\{-0.9^2\left(A\int_0^t 0.9^{-s}ds + Bc^x\int_0^t (0.9/c)^{-s}ds\right)\right\}
\end{aligned}$$

We can easily verify that for any positive constant $a > 0$, we have

$$\int_0^t a^{-s}ds = \frac{1 - a^{-t}}{\log(a)}.$$

Applying this, we then have

$${}_tp_{[x]} = \exp\left\{-0.9^2\left(A\,\frac{1 - 0.9^{-t}}{\log(0.9)} + Bc^x\,\frac{1 - (0.9/c)^{-t}}{\log(0.9/c)}\right)\right\}.$$

The result immediately follows by factoring out the term -0.9^{-t} from inside the parenthesis.