Exercise 4.14

$$\begin{aligned}
\bar{A}_x &= \int_0^\infty v^t {}_tp_x \mu_{x+t} dt \\
&= \sum_{k=0}^\infty \int_k^{k+1} v^t {}_tp_x \mu_{x+t} dt \\
& \quad \text{change variable of integration: } s = t - k \\
&= \sum_{k=0}^\infty \int_0^1 v^{s+k} {}_{s+k}p_x \mu_{x+s+k} ds \\
&= \sum_{k=0}^\infty v^k {}_kp_x \int_0^1 v^s {}_sp_{x+k} \mu_{x+k+s} ds
\end{aligned}$$

With constant force between integral ages assumption and according to the given, we have $\mu_{x+k+s} = \nu_{x+k}$ and ${}_sp_{x+k} = \exp(-\nu_{x+k}s)$ for any $0 \leq s < 1$. It follows that

$$\begin{aligned}
\bar{A}_x &= \sum_{k=0}^\infty v^k {}_kp_x \cdot \nu_{x+k} \int_0^1 e^{-(\delta+\nu_{x+k})s} ds \\
&= \sum_{k=0}^\infty v^k {}_kp_x \cdot \frac{\nu_{x+k}}{\delta + \nu_{x+k}} \left[1 - e^{-(\delta+\nu_{x+k})}\right] \\
&= \sum_{k=0}^\infty v^k {}_kp_x \cdot \frac{\nu_{x+k}}{\delta + \nu_{x+k}} \left[1 - e^{-\delta} e^{-\nu_{x+k}}\right] \\
&= \sum_{k=0}^\infty v^k {}_kp_x \cdot \frac{\nu_{x+k}\left(1 - vp_{x+k}\right)}{\delta + \nu_{x+k}}.
\end{aligned}$$