## Exercise 4.9

Start with

$$\begin{aligned}
(IA)^{1}_{x:\overline{n}|} &= \mathrm{E}\left[(K+1)v^{K+1}I(K<n)\right] \\
&= \mathrm{E}\left[(n+K+1-n)v^{K+1}I(K<n)\right] \\
&= (n+1)\mathrm{E}\left[v^{K+1}I(K<n)\right] - \mathrm{E}\left[(n-K)v^{K+1}I(K<n)\right] \\
&= (n+1)A^{1}_{x:\overline{n}|} - \sum_{k=0}^{n-1}(n-k)v^{k+1}{}_{k|}q_x .
\end{aligned}$$

Now, let us focus on the second term of the equation above. This term involves payments at the end of the year of death of $n$ if death occurs in the first year, $n-1$ if death occurs in the second year, decreasing by \$1 each year until maturity at which point the payment will only be \$1 if death occurs between $n-1$ and $n$. This policy is indeed called an $n$-year decreasing term insurance. Thus, we have

$$\begin{aligned}
(DA)^{1}_{x:\overline{n}|} &= \sum_{k=0}^{n-1}(n-k)v^{k+1}{}_{k|}q_x = \sum_{k=0}^{n-1}\sum_{s=0}^{n-k-1} v^{k+1}{}_{k|}q_x \\
&\quad \text{change the order of summation} \\
&= \sum_{s=0}^{n-1}\sum_{k=0}^{n-s-1} v^{k+1}{}_{k|}q_x = \sum_{s=0}^{n-1} A^{1}_{x:\overline{n-s}|} \\
&= A^{1}_{x:\overline{n}|} + A^{1}_{x:\overline{n-1}|} + \cdots + A^{1}_{x:\overline{1}|} \\
&\quad \text{reverse the order of the terms in the sum} \\
&= A^{1}_{x:\overline{1}|} + A^{1}_{x:\overline{2}|} + \cdots + A^{1}_{x:\overline{n}|} = \sum_{k=1}^{n} A^{1}_{x:\overline{k}|},
\end{aligned}$$

which proves the desired result. To interpret this result, it is better to rewrite the result as

$$(IA)^{1}_{x:\overline{n}|} + \sum_{k=1}^{n} A^{1}_{x:\overline{k}|} = (n+1)A^{1}_{x:\overline{n}|} .$$

Comparing the payments between the two sides of the equation, the increasing term pays 1, 2, increasing by 1 each year until maturity of $n$ years, and the decreasing term pays $n$, $n-1$, decreasing by 1 each year, until a payment of 1 remaining in the year before maturity. These two payments add up to payments of $n+1$ for $n$ years which is what would be required from the right side of the equation.