Exercise 5.13

Recall from **Exercise 2.9** that $\dfrac{d}{dx}{}_kp_x = {}_kp_x\big(\mu_x - \mu_{x+k}\big)$.

(a)

$$\begin{aligned}
\frac{d}{dx}\ddot{a}_x &= \sum_{k=0}^{\infty} v^k \cdot \frac{d}{dx}{}_kp_x \\
&= \sum_{k=0}^{\infty} v^k {}_kp_x\big(\mu_x - \mu_{x+k}\big) \\
&= \mu_x \sum_{k=0}^{\infty} v^k {}_kp_x - \sum_{k=0}^{\infty} v^k {}_kp_x \mu_{x+k} \\
&= \mu_x \ddot{a}_x - \sum_{k=0}^{\infty} v^k {}_kp_x \mu_{x+k}
\end{aligned}$$

(b) Similar to (a), we can show that

$$\frac{d}{dx}\ddot{a}_{x:\overline{n|}} = \mu_x \ddot{a}_{x:\overline{n|}} - \sum_{k=0}^{n-1} v^k {}_kp_x \mu_{x+k}$$