Exercise 5.2

(a) Y is the present value random variable associated with an annuity that pays continuously at the rate of \$1 per year where the payment starts at the moment of death of (x) continuing until the end of year n, provided (x) dies before the end of n years.

(b) Notice that we can express

$$\begin{aligned} v^T \bar{a}_{\overline{n-T|}} &= v^T \frac{1-v^{n-T}}{\delta} = \frac{v^T - v^n}{\delta} \\ &= \frac{1-v^n}{\delta} - \frac{1-v^T}{\delta} = \bar{a}_{\overline{n|}} - \bar{a}_{\overline{T|}} \end{aligned}$$

Thus, we can write $Y = \bar{a}_{\overline{n|}} - Y_1$ where

$$Y_1 = \begin{cases} \bar{a}_{\overline{T|}}, & \text{for } T \le n \\ \bar{a}_{\overline{n|}}, & \text{for } T > n \end{cases}$$

is the present value random variable for an n-year temporary continuous life annuity on (x). The result immediately follows:

$$\mathrm{E}[Y] = \bar{a}_{\overline{n|}} - \bar{a}_{x:\overline{n|}}.$$

(c) The first term, $\bar{a}_{\overline{n|}}$, pays a guaranteed benefit of \$1 per year for n years while the second term, $\bar{a}_{x:\overline{n|}}$, pays an annuity of \$1 per year while (x) is alive within the next n years. The difference therefore is the Family Income Benefit that pays \$1 starting at the moment of death of (x) continuing until the end of n years, provided (x) does not survive to reach $x+n$.