Exercise 6.12

Let P be the annual premium determined according to the equivalence principle. Then

$$P = \frac{A_x}{\ddot{a}_x}$$

and

$$L_0 = v^{K+1} - P\ddot{a}_{\overline{K+1}|} = \left(1 + \frac{P}{d}\right) v^{K+1} - \frac{P}{d}.$$

Let P^* be the annual premium determined according to $\mathrm{E}[L_0^*] = -0.50$. Then

$$P^* = \frac{A_x + 0.5}{\ddot{a}_x}$$

and

$$L_0^* = \left(1 + \frac{P^*}{d}\right) v^{K+1} - \frac{P^*}{d}.$$

The ratio of the variances of the respective loss random variables is therefore given by

$$\begin{aligned}
\frac{\mathrm{Var}\left[L_0^*\right]}{\mathrm{Var}\left[L_0\right]} &= \frac{\left[1 + (P^*/d)\right]^2 \mathrm{Var}\left[v^{K+1}\right]}{\left[1 + (P/d)\right]^2 \mathrm{Var}\left[v^{K+1}\right]} \\
&= \left(\frac{P^* + d}{P + d}\right)^2 = \left(\frac{A_x + 0.5 + d\ddot{a}_x}{A_x + d\ddot{a}_x}\right)^2 \\
&\quad \text{since } A_x + d\ddot{a}_x = 1 \\
&= (1.5)^2
\end{aligned}$$

It follows therefore that

$$\mathrm{Var}\left[L_0^*\right] = (1.5)^2\, \mathrm{Var}\left[L_0\right] = (1.5)^2(0.75) = 1.6875.$$