Exercise 6.17

(a) $P = 30000\, a_{[60]} = 30000\, (\ddot{a}_{[60]} - 1) = 30000(14.913 - 1) = 417,390$

(b) We can indeed write the present value of profit as

$$\text{Profit} = -L_0 = P - 30000 a_{\overline{K|}}.$$

The event profit is positive is equivalent to the event

$$P > 30000 a_{\overline{K|}}$$

This event is equivalent to

$$K < -\frac{1}{\delta} \log\left(1 - \frac{Pi}{30000}\right) = -\frac{1}{\log(1.05)} \log\left(1 - \frac{(417390)(0.05)}{30000}\right) = 24.38149.$$

Thus, we have

$$\begin{aligned} \Pr[\text{Profit} > 0] &= \Pr[K < 24.38149] = \Pr[K \leq 24] \\ &= 1 - {}_{25}p_{[60]} = 1 - \frac{\ell_{85}}{\ell_{[60]}} = \frac{61184.88}{96568.13} = 0.3664071. \end{aligned}$$

(c) The variance of the present value of profit is

$$\begin{aligned} \text{Var}[\text{Profit}] &= \text{Var}[L_0] = \left(\frac{30000}{d}\right)^2 \left[{}^2A_{[60]} - \left(A_{[60]}\right)^2\right] \\ &= \left(\frac{30000}{0.05/1.05}\right)^2 \left[0.10781 - (0.28984)^2\right] = 9447321159, \end{aligned}$$

so that the standard deviation is

$$\text{SD}[\text{Profit}] = 97197.33.$$

(d) For a 1000 such annuities, the total profit is

$$\text{TP} = -\sum_{i=0}^{1000} L_{0,1} = -\sum_{i=0}^{1000} \left(P - 30000 a_{\overline{K|}}\right)$$

where its mean is

$$\text{E}[\text{TP}] = -1000(P - 417390)$$

and its standard deviation is

$$\text{SD}[\text{TP}] = \sqrt{1000} \times 97197.33.$$

Thus we require

$$\Pr[\mathrm{TP} > 0] = 0.95 \iff \Pr\left[Z > \frac{1000(P - 417390)}{\sqrt{1000}(97197.33)}\right] = 0.95$$

Solving for P from

$$\frac{1000}{\sqrt{1000}} \frac{P - 417390}{97197.33} = 1.645,$$

we get

$$P = 417390 + 1.645 \times \frac{\sqrt{1000}}{1000} 97197.33 = 422446.2.$$