Exercise 6.2

(a) Let P be the net single premium. The loss-at-issue random variable can be written as

$$L_0 = \text{PVFB}_0 - \text{PVFP}_0 = 1000000v^{K+1}I(K<5) - P$$

(b) Solving for P, we get

$$\begin{aligned} P &= 1000000 \times \bar{A}^{\,1}_{[40]:\overline{5}|} \\ &= 1000000 \times \frac{i}{\delta}\left(A_{[40]} - {}_5E_{[40]}A_{45}\right) \\ &= 1000000 \times \frac{0.05}{\log(1.05)}[0.12097 - 0.78121(0.15161)] = 2593.506 \end{aligned}$$

(c) The event $L_0 < 0$ is equivalent to the event

$$1000000v^{K+1}I(K<5) - P < 0.$$

When $K = 4$, we can verify that $L_0 = 780932.7$ so that $K > 5$. Therefore, we have

$$\Pr[L_0<0] = \Pr[K>5] = {}_5p_{[40]} = \frac{\ell_{45}}{\ell_{[40]}} = \frac{99033.94}{99327.82} = 0.9970413.$$

The contract makes a profit only if the person select age 40 will survive another 5 years, or will never die during the term of the policy.