Exercise 6.4

(a) Let P be the required annual benefit premium. The loss-at-issue random variable can be written as

$$\begin{aligned} L_0 &= \text{PVFB}_0 - \text{PVFP}_0 \\ &= \begin{cases} 1000v^{K+1} - P\ddot{a}_{\overline{K+1}|}, & \text{for } K = 0,1,\ldots,9 \\ -P\ddot{a}_{\overline{10}|}, & \text{for } K = 10,11,\ldots \end{cases} \\ &= 1000v^{K+1}I(K<10) - P\big[\ddot{a}_{\overline{K+1}|}I(K<10) + \ddot{a}_{\overline{10}|}I(K\geq 10)\big] \end{aligned}$$

(b) According to the equivalence principle, we set $\mathrm{E}[L_0] = 0$ to solve for P:

$$P = 1000 \times \frac{A^{\;\;1}_{[50]:\overline{10}|}}{\ddot{a}_{[50]:\overline{10}|}},$$

where

$$\begin{aligned} {}_{10}E_{[50]} &= v^{10}\,{}_{10}p_{[50]} = v^{10}\frac{\ell_{60}}{\ell_{[50]}} = (1.05)^{-10}\frac{96634.14}{98552.51} = 0.6019631, \\ \ddot{a}_{[50]:\overline{10}|} &= \ddot{a}_{[50]} - {}_{10}E_{[50]}\,\ddot{a}_{60} = 17.02835 - 0.6019631(14.90407) = 8.056649, \text{ and} \\ A^{\;\;1}_{[50]:\overline{10}|} &= A_{[50]:\overline{10}|} - {}_{10}E_{[50]} = \big(1 - d\ddot{a}_{[50]:\overline{10}|}\big) - {}_{10}E_{[50]} \\ &= \big[1 - (1-(1.05)^{-1})(8.056649)\big] - 0.6019631 = 0.01438689. \end{aligned}$$

Therefore, we have

$$P = 1000 \times \frac{0.01438689}{8.056649} = 178.5717.$$