Exercise 6.8

Denote by G the required annual premium.

First, consider the actuarial present value (APV) of the benefits (note that there is no mention of any type of approximation, and hence, the APV has to be computed from basic principle):

$$\begin{aligned}\text{APV(benefits)} &= 200000 \times \sum_{k=0}^{\infty}\sum_{j=0}^{11}(1.015)^k v^{k+\frac{j+1}{12}} {}_{k+\frac{j}{12}}p_{[40]}\ {}_{\frac{1}{12}}q_{[40]+k+\frac{j}{12}} \\ &= 200000 \times \sum_{k=0}^{\infty}(1.015v)^k \sum_{j=0}^{11} v^{\frac{j+1}{12}} {}_{k+\frac{j}{12}}p_{[40]}\ {}_{\frac{1}{12}}q_{[40]+k+\frac{j}{12}},\end{aligned}$$

where it can be verified that for $j = 0, 1, \ldots, 23$,

$${}_{\frac{1}{12}}p_{[40]+\frac{j}{12}} = \exp\left\{(0.9)^{2-\frac{j+1}{12}}\left[\frac{1-0.9^{1/12}}{\log(0.9)}A + Bc^{40+(j/12)}\frac{c^{1/12}-0.9^{1/12}}{\log(0.9/c)}\right]\right\}$$

and that for $j = 24, 25, \ldots$, we have

$${}_{\frac{1}{12}}p_{40+\frac{j}{12}} = \exp\left[-\frac{1}{12}A - Bc^{40+(j/12)}\frac{c^{1/12}-1}{\log(c)}\right].$$

The following R code calculates this APV based on the Standard Select Survival Model with $i = 5\%$:

```
# Makeham parameters:
A <- .00022
B <- 2.7*10^(-6)
c <- 1.124
m <- 12
x <- seq(40,131,1/m)
kpmx <- rep(0,length(x))
s <- 0:(2*m-1)
temp1 <- (0.9)^(2-(s+1)/m)
temp2 <- (1-(0.9)^(1/m))*A/log(0.9)
temp3 <- B*c^(x[1]+s/m)*(c^(1/m) - (0.9)^(1/m))/log(0.9/c)
kpmx[1:length(s)] <- exp(temp1 * (temp2+temp3))
j <- (2*m):(length(x)-1)
temp4 <- (B/log(c))*c^(x[1]+j/m)*(c^(1/m)-1)
len <- (length(s)+1):length(x)
kpmx[len] <- exp(-A/m - temp4)
kqmx <- 1-kpmx
kpmx <- cumprod(c(1,kpmx[-length(kpmx)]))
int <- .05
v <- 1/(1+int)
vj <- v^((1:m)/m)
x2 <- 40:131
APV.40 <- 0
```

```
k <- 0
while (k<(length(x2)-1)) {
j <- 0
temp <- 0
while (j < m) {
j <- j+1
temp <- temp + vj[j] * kpmx[k*m+j] * kqmx[k*m+j]
}
APV.40 <- APV.40 + (1.015*v)^k * temp
k <- k+1
}
APV.40 <- 200000*APV.40
```

This produces the results:

```
> APV.40
[1] 44586.36
```

Next, we consider the APV of the expenses:

$$\text{APV(expenses)} = 0.575G + 0.025G\ddot{a}_{[40]:\overline{25}|} + 5\sum_{k=0}^{24}(1.06)^k v^k {}_k p_{[40]}$$

where the summation term on the RHS was computed using the R code:

```
# geometrically increasing temporary life annuity-due
kp40 <- rep(0,25)
temp1 <- ((1-.9)/log(0.9))*A
temp2 <- ((c-0.9)/log(0.9/c))*B*c^(40)
kp40[1] <- exp(0.9*(temp1+temp2))
kp40[2] <- exp(((1-.9^2)/log(0.9))*A + ((c^2 - 0.9^2)/log(0.9/c))*B*c^(40))
k <- 2
while (k<length(kp40)) {
k <- k+1
temp1 <- A*(k-2)
temp2 <- ((c^(k-2) - 1)/log(c))*B*c^(40+2)
kp40[k] <- kp40[2]*exp(-temp1-temp2)
}
v <- 1.06/1.05
vk <- v^(0:(length(kp40)-1))
kp40 <- c(1,kp40[-length(kp40)])
ann.4025s <- sum(vk*kp40)
```

This produces the results:

```
> ann.4025s
[1] 27.66275
```

Finally, the APV of the premiums can be expressed as

$$\text{APV(premiums)} = G\ddot{a}_{[40]:\overline{25}|},$$

where using the Standard Select Survival Model at 5%, we have

$$\begin{aligned}
\ddot{a}_{[40]:\overline{25}|} &= \ddot{a}_{[40]} - v^{25}\frac{\ell_{65}}{\ell_{[40]}}\ddot{a}_{65} \\
&= 18.45956 - (1.05)^{-25} \cdot \frac{94579.73}{99327.82} \cdot 13.54979 \\
&= 14.64954
\end{aligned}$$

Applying the equivalence principle, we set APV(premiums) = APV(benefits) + APV(expenses) which leads us to

$$\begin{aligned}
G &= \frac{\text{APV(benefits)} + 5\sum_{k=0}^{24}(1.06)^k v^k {}_kp_{[40]}}{0.975\ddot{a}_{[40]:\overline{25}|} - 0.575} \\
&= \frac{44586.36 + 5(27.66275)}{0.975(14.64954) - 0.575} \\
&= 3262.597
\end{aligned}$$