Michigan State University

STT 456 - Actuarial Models II

Spring 2015 semester

Homework

due Friday, 5:00 pm, February 6, 2015

Please follow the instructions below:

Please do all four (4) questions and their subparts. The `Illustrative Life Table` is attached for your use.

Return this page with your signature.

Submit your work to our graduate assistant, Ed Cruz, at C505 Wells.

Write your name and section number at the spaces provided:

Name: Suggested Solutions **Section:** ____________________

I certify that this is my own work, and that I have not copied the work of another student.

Signature: ____________________________ **Date:** ____________________

1. (35 points) Cest-la-Vie Life Insurance Company decided to sell fully discrete whole life insurance of 1,000 to individuals age 50, something it has not done before.

   Actuaries at Cest-la-Vie have determined that:

   - Mortality is expected to follow the `Illustrative Life Table` and premiums are calculated at interest rate $i = 0.06$.
   - All the policyholders will have independent future lifetimes.

   (a) [5 points] Suppose P is the annual premium for each policy calculated based on the equivalence principle. Give an expression for the loss-at-issue random variable for a single policy, and calculate P.

   (b) [10 points] Suppose Cest-la-Vie sells n of these policies with each policy paying an annual premium of π_n. Determine the expected value and the variance of the loss-at-issue for such a policy in terms of π_n.

   (c) [15 points] Based on the normal approximation, the premium π_n is determined such that the probability of the aggregate loss from all policies is positive is 0.05. Express π_n in terms of n only.

   (d) [3 points] Now, finally, calculate $\pi = \lim_{n\to\infty} \pi_n$ and show that $P = \pi$, where P is the value you calculated in part (a).

   (e) [2 points] Explain in words why $P = \pi$. What is the intuition behind this equality?

(a) Loss at issue $L_0 = 1000 v^{K+1} - P \ddot{a}_{\overline{K+1}|}$

$$E[L_0] = 0 \Rightarrow \quad P = \frac{1000\, E[v^{K+1}]}{E[\ddot{a}_{\overline{K+1}|}]} = 1000 \frac{A_{50}}{\ddot{a}_{50}} = \frac{1000(.24905)}{13.2668} = \underline{\underline{18.77242}}$$

(b) Here, $L_0 = 1000 v^{K+1} - \pi_n \ddot{a}_{\overline{K+1}|}$ $\quad \frac{1-v^{K+1}}{d}$ which can also be written as

$$= \left(1000 + \frac{\pi_n}{d}\right) v^{K+1} - \pi_n/d$$

$$E[L_0] = 1000 A_{50} - \pi_n \ddot{a}_{50} = 1000(.24905) - 13.2668\, \pi_n$$
$$= 249.05 - 13.2668\, \pi_n$$

and

$d = .06/1.06 = 6/106 = 3/53$

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$$Var[L_0] = \left(1000 + \frac{\Pi_n}{d}\right)^2 \underbrace{Var[v^{K+1}]}_{{}^2A_{50} - (A_{50})^2 \, = \, \underbrace{.09476 - (.24905)^2}_{.0327341}}$$

$$= \left(1000 + \frac{53}{3}\Pi_n\right)^2 (.0327341)$$

(d) Aggregate loss $= L = \sum_{i=1}^{n} L_{0,i}$ so that

$$E[L] = n E[L_0] = (249.05 - 13.2668\Pi_n)\, n$$

and

$$Var[L] = n\, Var[L_0] = \left(1000 + \frac{53}{3}\Pi_n\right)^2 (.0327341)\, n$$

$$P_r[L > 0] = P_r\left[\frac{L - E[L]}{\sqrt{Var[L]}} \cong Z = \frac{-(249.05 - 13.2668\Pi_n)n}{\sqrt{\left(1000 + \frac{53}{3}\Pi_n\right)^2 (.0327341)n}} > 0\right] = .05$$

$$\Rightarrow \quad \frac{-(249.05 - 13.2668\,\Pi_n)\,n}{\left(1000 + \frac{53}{3}\Pi_n\right)\sqrt{(.0327341)n}} = 1.645$$

Solving for Π_n, we get

$$\Pi_n = \frac{1645\sqrt{.0327341}\cdot\sqrt{n} + 249.05\,n}{13.2668\,n - 1.645\left(\frac{53}{3}\right)\sqrt{.0327341}\,\sqrt{n}}$$

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(d) Divide both numerator & denominator by $\sqrt{n}$, we get

$$\Pi_n = \frac{1645\sqrt{.0327341}/\sqrt{n} + 249.05}{13.2668 - 1.645\left(\frac{53}{3}\right)\sqrt{.0327341}/\sqrt{n}}$$

Thus, we see that as $n \to \infty$ $1/\sqrt{n} \to 0$, so that

$$\Pi = \lim_{n\to\infty} \Pi_n = \frac{249.05}{13.2668} = 18.77242$$ equal to P in (a).

(e) While it is true that the variance increases with n, the average of these variances decrease with n.

Intuitively this means that the more there are in the pool, the more predictable the loss is going to be.

2. (15 points) A discrete whole life annuity pays 1,000 at the beginning of each year that life (65) is alive. You are given:

- Standard mortality is based on the `Illustrative Life Table`.
- $i = 0.06$

The standard force of mortality is superscripted by "ILT". A substandard risk has a force of mortality given by

$$\mu^{s}_{65+t} = \begin{cases} \mu^{\text{ILT}}_{65} + 0.03, & \text{for } t = 0 \\ \mu^{\text{ILT}}_{66} + 0.01, & \text{for } t = 1 \\ \mu^{\text{ILT}}_{65+t}, & \text{for } t \geq 2 \end{cases}$$

(a) [5 points] Calculate the actuarial present value of this annuity for a standard life (65).

(b) [8 points] Calculate the actuarial present value of this annuity for a substandard life (65).

(c) [2 points] Explain in words why the value in (b) is smaller than that in (a).

(a) $APV = 1000\,\ddot{a}_{65} = 1000(9.8969) = \underline{\underline{9{,}896.90}}$

(b) $APV = 1000\,\ddot{a}^{s}_{65}$ — substandard

$$= 1000\left[1 + v\,p^{s}_{65} + v^2\,p^{s}_{65}\,p^{s}_{66}\,\ddot{a}^{s}_{67}\right]$$

Because mortality are the same at age 67 & later, the annuity value are the same at age 67, so that

$$= 1000\left[1 + \frac{1}{1.06}\,p^{\text{ILT}}_{65}\,e^{-.03} + \frac{1}{1.06^2}\,p^{\text{ILT}}_{65}\,p^{\text{ILT}}_{66}\,e^{-.03}e^{-.01}\,\ddot{a}_{67}\right]$$

$$p^{\text{ILT}}_{65} = \left(1 - \frac{21.32}{1000}\right) \qquad p^{\text{ILT}}_{66} = \left(1 - \frac{23.29}{1000}\right) \qquad \ddot{a}_{67} = 9.3726$$

$$= 1000\left[1 + \frac{1}{1.06}\left(1 - \frac{21.32}{1000}\right)e^{-.03} + \frac{1}{1.06^2}\left(1 - \frac{21.32}{1000}\right)\left(1 - \frac{23.29}{1000}\right)e^{-.04}\,9.3726\right]$$

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From (b), we get APV = 9556.951.

which is smaller than that in (b)

(c) In general, the higher mortality for substandard risks lowers the life expectation; this reduces the value of the annuity because fewer payments are expected to be made!

3. (35 points) For a fully discrete whole life insurance of 10,000 on (40), you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

(a) [10 points] Calculate the net premium reserve at the end of year 10.

(b) [10 points] Suppose expenses, incurred at the beginning of the year, consist of 2.1 in all years. Calculate the gross premium reserve at the end of year 10.

(c) [10 points] Suppose instead of a constant expense each year, expenses are 10.0 in the first year and 1.5 in subsequent years. Calculate the gross premium reserve at the end of year 10.

(d) [2 points] Explain in words why the values in (a) and (b) are the same.

(e) [3 points] Explain in words why the values in (a) and (c) are different.

(a) First, compute premium $P = 10{,}000 \dfrac{\overset{.16132}{A_{40}}}{\underset{14.8166}{\ddot{a}_{40}}} = \underline{\underline{108.8779}}$

$$ {}_{10}V^n = 10{,}000 A_{50} - P\ddot{a}_{50}$$
$$= 10{,}000(.24905) - (108.8779)(13.2668) = \underline{\underline{1{,}046.039}}$$

(b) Premium is $G\ddot{a}_{40} = 10{,}000 A_{40} + 2.1\ddot{a}_{40}$

$$G = 10{,}000 \frac{A_{40}}{\ddot{a}_{40}} + 2.1 = P + 2.1 = \underline{\underline{110.9779}}$$

$${}_{10}V^g = 10{,}000 A_{50} + \cancel{2.1\ddot{a}_{50}} - (P + \cancel{2.1})\ddot{a}_{50}$$
$$= \underbrace{10{,}000 A_{50} - P\ddot{a}_{50}}_{\text{exactly as in (c)}} = \underline{\underline{1{,}046.039}}$$

(c) Premium here is $G\ddot{a}_{40} = 10{,}000 A_{40} + 8.5 + 1.5\ddot{a}_{40}$

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so that $G = \underbrace{\frac{10{,}000 A_{40}}{\ddot{a}_{40}}}_{P} + 1.5 \frac{\cancel{\ddot{a}_{40}}}{\cancel{\ddot{a}_{40}}} + \frac{8.5}{\ddot{a}_{40}}$

$$= \underbrace{\underset{108.8779}{P} + 1.5 + \frac{8.5}{\underset{14.8166}{\ddot{a}_{40}}}}_{\text{here we see } G \text{ is } P + \text{renewal} + \text{amortization of extra 1st yr expense}} = \underline{\underline{110.9516}}$$

$${}_{10}V^g = 10{,}000 A_{50} + \cancel{1.5 \ddot{a}_{50}} - \left(P + \cancel{1.5} + \frac{8.5}{\ddot{a}_{40}}\right) \ddot{a}_{50}$$

$$= \underbrace{10{,}000 A_{50} - P \ddot{a}_{50}}_{{}_{10}V^n} - \frac{8.5}{\ddot{a}_{40}} \ddot{a}_{50} = \underline{\underline{1038.428}}$$

(d) When expenses are level/constant each year, the gross premium is simply the net premium plus the annual expense. This means expenses are recovered from premiums as soon as they are incurred. This has no effect then on reserves.

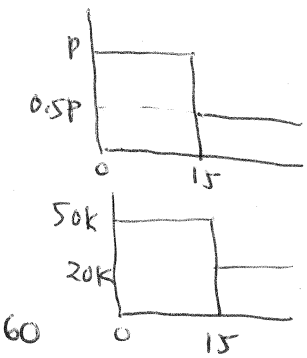
(e) For a larger expense in the first year, the difference is amortized (or spread) over the life of the policy. In this case, gross premium is augmented by this amortization & is therefore recovered in future premiums. This recovery allows insurer to hold lower reserves!

4. (15 points) For a special fully discrete whole life insurance on (45), you are given:

- The death benefit is 50,000 in the first 15 years, and 25,000 thereafter.
- The net annual premium is P in the first 15 years, and reduces to $0.5P$ in subsequent years.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Using recursive formulas, calculate the net premium reserves in years 0, 1 and 2.

First calculate P.



$$APV(FP_0) = APV(FB_0) \Rightarrow$$

$$P\ddot{a}_{45} - 0.5P\,{}_{15}E_{45}\,\ddot{a}_{60} = 50{,}000\,A_{45} - 25{,}000\,{}_{15}E_{45}\,A_{60}$$

Solving for P, we get

$$P = \frac{50{,}000\left(A_{45} - .5\,{}_{15}E_{45}\,A_{60}\right)}{\ddot{a}_{45} - .5\,{}_{15}E_{45}\,\ddot{a}_{60}}$$

$A_{45} = .20120$

$A_{60} = .36913$

${}_{15}E_{45} = {}_{10}E_{45}\,{}_{5}E_{55}$

$= \underbrace{.52652(.70810)}_{.3728288}$

$\ddot{a}_{45} = 14.1121$

$\ddot{a}_{60} = 11.1454$

plug values, we get

$$P = \underline{\underline{550.0417}}$$

Starting with ${}_0V = 0$, we have

$${}_1V = \frac{({}_0V + P)(1+i) - 50{,}000\,q_{45}}{1 - q_{45}}$$

$$= \frac{550.0417(1.06) - 50{,}000\,(4/1000)}{1 - 4/1000} = \underline{\underline{384.5826}}$$

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$$_2V = \frac{(_1V + P)(1+i) - 50{,}000\, q_{46}}{1 - q_{46}}$$

$$= \frac{(384.5826 + 550.0417)(1.06) - 50{,}000\left(4.31/1000\right)}{1 - 4.31/1000}$$

$$= \underline{\underline{778.5573}}$$

NORMAL DISTRIBUTION TABLE

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z<z)
The value of z to the first decimal is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Values of z for selected values of Pr(Z<z)							
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000{}_5E_x$	$1000{}_{10}E_x$	$1000{}_{20}E_x$	x
0	10,000,000	20.42	16.8010	49.00	25.92	728.54	541.95	299.89	0
5	9,749,503	0.98	17.0379	35.59	8.45	743.89	553.48	305.90	5
10	9,705,588	0.85	16.9119	42.72	9.37	744.04	553.34	305.24	10
15	9,663,731	0.91	16.7384	52.55	11.33	743.71	552.69	303.96	15
20	9,617,802	1.03	16.5133	65.28	14.30	743.16	551.64	301.93	20
21	9,607,896	1.06	16.4611	68.24	15.06	743.01	551.36	301.40	21
22	9,597,695	1.10	16.4061	71.35	15.87	742.86	551.06	300.82	22
23	9,587,169	1.13	16.3484	74.62	16.76	742.68	550.73	300.19	23
24	9,576,288	1.18	16.2878	78.05	17.71	742.49	550.36	299.49	24
25	9,565,017	1.22	16.2242	81.65	18.75	742.29	549.97	298.73	25
26	9,553,319	1.27	16.1574	85.43	19.87	742.06	549.53	297.90	26
27	9,541,153	1.33	16.0873	89.40	21.07	741.81	549.05	297.00	27
28	9,528,475	1.39	16.0139	93.56	22.38	741.54	548.53	296.01	28
29	9,515,235	1.46	15.9368	97.92	23.79	741.24	547.96	294.92	29
30	9,501,381	1.53	15.8561	102.48	25.31	740.91	547.33	293.74	30
31	9,486,854	1.61	15.7716	107.27	26.95	740.55	546.65	292.45	31
32	9,471,591	1.70	15.6831	112.28	28.72	740.16	545.90	291.04	32
33	9,455,522	1.79	15.5906	117.51	30.63	739.72	545.07	289.50	33
34	9,438,571	1.90	15.4938	122.99	32.68	739.25	544.17	287.82	34
35	9,420,657	2.01	15.3926	128.72	34.88	738.73	543.18	286.00	35
36	9,401,688	2.14	15.2870	134.70	37.26	738.16	542.11	284.00	36
37	9,381,566	2.28	15.1767	140.94	39.81	737.54	540.92	281.84	37
38	9,360,184	2.43	15.0616	147.46	42.55	736.86	539.63	279.48	38
39	9,337,427	2.60	14.9416	154.25	45.48	736.11	538.22	276.92	39
40	9,313,166	2.78	14.8166	161.32	48.63	735.29	536.67	274.14	40
41	9,287,264	2.98	14.6864	168.69	52.01	734.40	534.99	271.12	41
42	9,259,571	3.20	14.5510	176.36	55.62	733.42	533.14	267.85	42
43	9,229,925	3.44	14.4102	184.33	59.48	732.34	531.12	264.31	43
44	9,198,149	3.71	14.2639	192.61	63.61	731.17	528.92	260.48	44
45	9,164,051	4.00	14.1121	201.20	68.02	729.88	526.52	256.34	45
46	9,127,426	4.31	13.9546	210.12	72.72	728.47	523.89	251.88	46
47	9,088,049	4.66	13.7914	219.36	77.73	726.93	521.03	247.08	47
48	9,045,679	5.04	13.6224	228.92	83.06	725.24	517.91	241.93	48
49	9,000,057	5.46	13.4475	238.82	88.73	723.39	514.51	236.39	49
50	8,950,901	5.92	13.2668	249.05	94.76	721.37	510.81	230.47	50
51	8,897,913	6.42	13.0803	259.61	101.15	719.17	506.78	224.15	51
52	8,840,770	6.97	12.8879	270.50	107.92	716.76	502.40	217.42	52
53	8,779,128	7.58	12.6896	281.72	115.09	714.12	497.64	210.27	53
54	8,712,621	8.24	12.4856	293.27	122.67	711.24	492.47	202.70	54
55	8,640,861	8.96	12.2758	305.14	130.67	708.10	486.86	194.72	55
56	8,563,435	9.75	12.0604	317.33	139.11	704.67	480.79	186.32	56
57	8,479,908	10.62	11.8395	329.84	147.99	700.93	474.22	177.53	57
58	8,389,826	11.58	11.6133	342.65	157.33	696.85	467.12	168.37	58
59	8,292,713	12.62	11.3818	355.75	167.13	692.41	459.46	158.87	59
60	8,188,074	13.76	11.1454	369.13	177.41	687.56	451.20	149.06	60
61	8,075,403	15.01	10.9041	382.79	188.17	682.29	442.31	139.00	61
62	7,954,179	16.38	10.6584	396.70	199.41	676.56	432.77	128.75	62
63	7,823,879	17.88	10.4084	410.85	211.13	670.33	422.54	118.38	63
64	7,683,979	19.52	10.1544	425.22	223.34	663.56	411.61	107.97	64
65	7,533,964	21.32	9.8969	439.80	236.03	656.23	399.94	97.60	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000{}_5E_x$	$1000{}_{10}E_x$	$1000{}_{20}E_x$	x
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90
91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110