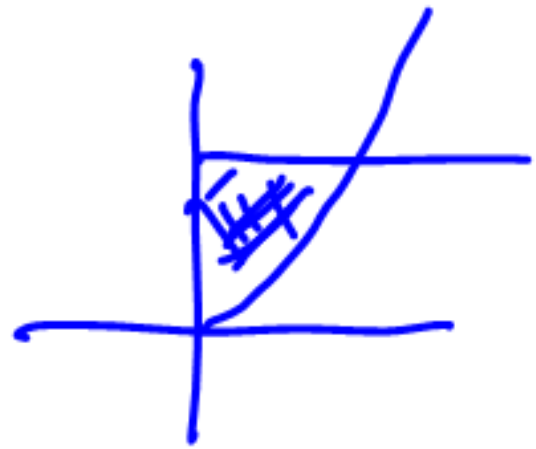
Universal Life Insurance [†]

Lecture: Weeks 11-12



[†] Thanks to my friend J. Dhaene, KU Leuven, for ideas here drawn from his notes.

Chapter summary

- What is a Universal Life (UL) insurance product?
  - when compared to traditional insurance products
  - key features e.g. flexibility, transparency
- The emerging cash flows in a UL policy
- Additional features/provisions:
  - no-lapse guarantee
  - corridor factor provisions
- Materials on:
  - Chapter 13: sections 13.4 and 13.5

Drawbacks of traditional life insurance

There are many identified drawbacks of traditional products that make them lose its attractiveness over the years:

- the lack of flexibility
  - premiums, benefits (death, withdrawals, survival)
- complicated, not straightforward for consumers to understand
- the lack of transparency
  - consumer does not have any idea how much is being saved (for say cash value), how much is used to fund benefits

Main features of Universal Life (UL) products

This led to the introduction of UL policies designed for consumers who wish for:

- increased flexibility
  - adjust premiums and benefits within certain constraints (to avoid selection issue)
- "unbundled" feature
  - a more transparent separation of the benefit and savings components
  - a similar notion to "buy term, invest the difference"
- the investment feature
  - interest is credited to the account on a periodic basis, with some minimum interest guarantees
  - variations to traditional UL, e.g. Variable UL, Equity Indexed UL, allow investment options for opportunity to gain more on investment

The account value

Consider a UL policy issued to (x) at time 0, with unit of time as year. For each time interval then between $(k-1, k)$ for $k = 1, 2, \ldots$:

- the policyholder pays (or deposits) a premium π_{k-1} at the beginning of the period,
- the insurance company assesses the following fees or charges:
  - f, a percent of premium charge,
  - e, an expense charge to cover administrative and related expenses, and
  - COI, the Cost of Insurance charge to cover death benefits.
- interest i_k^c is credited for the period.

Note that the charges f, e, and COI may vary with time (and possibly issue age).

$$AV_k = \left(AV_{k-1} + \pi_{k-1} - f\,\pi_{k-1} - e - COI\right)\left(1+i_k^c\right)$$

AV_{k-1} ↓ beg acct

π_{k-1} ↓ prem

$f\,\pi_{k-1}$ ↓ % exp

$$\approx \left(AV_{k-1} + \pi_{k-1}(1-f) - e - \frac{(DB_k - AV_k)}{1+i^*} \cdot coi\text{-}rate_k\right)\left(1+i_k^c\right)$$

Calculation of the account value

The account value (sometimes called account balance) then at the end of year k is equal to

$$\mathsf{AV}_k = [\mathsf{AV}_{k-1} + \pi_{k-1}(1-f) - e - \mathsf{COI}] \times (1 + i_k^c),$$

where

$$\mathsf{COI} = \frac{\mathsf{DB}_k - \mathsf{AV}_k}{1 + i_k^q}(coi\_rate)$$

and

- DB_k is the death benefit payable at the end of the year,
- i_k^q is the interest rate per period used to discount the net amount at risk in the COI calculation, which if not stated, one could assume equal to i_k^c, and
- coi_rate is the cost of insurance rate (that is, the cost of insurance per dollar of benefit).

Some helpful remarks

- The cost of insurance rate is typically expressed as a percentage of the applicable mortality rate at the attained age of the insured:
  - q_{x+k-1} is the (annual) rate of mortality for the period $(k-1, k)$
- At policy surrender (or withdrawal) prior to policy maturity, the surrender value is the account value reduced by a surrender charge.
  - The surrender value is sometimes referred to as the cash value.
  - The surrender charge is assessed to recoup any unrecovered acquisition expenses.
  - The cash value cannot be negative so that: $\text{CV} = \max(\text{AV} - \text{SC}, 0)$

Death benefit options

DB = AV + ADD

Broadly speaking, the total death benefit is the policy's account value plus an additional death benefit (ADB).

- **Type A**: level total death benefit
  - As the account value then increases (because of premium additions and interest credited), the ADB decreases.
- **Type B**: level ADB
  - Here, the total death benefit is the AV plus the chosen level ADB.
- These are subject to the corridor factor requirement.
  - By law, the policy must be considered an insurance contract and this is tested using the ratio $\frac{\text{AV} + \text{ADB}}{\text{AV}}$ called the corridor factor.
  - In the US, this factor is about 2.5 times up until age 40, decreasing gradually to 1.05 times by age 90, and then to 1.0 times by age 95.

DB = AV + "Add. DB"

DB - AV = "Add. DB"

Type A varies

Type B fixed

DB

end of year

$$AV_k = \left(AV_{k-1} + \Pi_{k-1}(1-f) - e - COI\right)(1+i^c)$$

$$COI = \frac{(DB_k - AV_k)}{1+i^g} \cdot \%g_{k-1}$$

$$i^c = i^g$$

$$= \left(AV_{k-1} + \Pi_{k-1}(1-f) - e\right)(1+i^c) - (DB_k - AV_k) * \%g_{k-1}$$

Type A Total DB = ADB + AV Type B

Total DB is constant

$i^c = i^q = i$

ADB is constant

110 - 120%

$$AV_k = \left(AV_{k-1} + \Pi_{k-1}(1-f) - e\right)(1+i) - (DB_k - AV_k)\,\beta\%\,q_{k-1}$$

$$AV_k(1 - \%q_{k-1}) = -DB_k\,\%q_{k-1}$$

$$AV_k = \frac{-DB_k\,\%q_{k-1}}{1 - \%q_{k-1}}$$

Type B: $DB_k - AV_k = ADB$ is fixed

$$AV_k = \left(AV_{k-1} + \Pi_{k-1}(1-f) - e\right)(1+i) - ADB \cdot \%q_{k-1}$$

corridor factor $\frac{AV + ADB}{AV}$ meet minimum standards!

Additional features

- no lapse guarantee
  - Death benefit coverage continues even if AV falls to zero, subject to paying a pre-specified minimum premium at each premium date.
- policy loans
  - Most UL policies would allow policyholder to borrow with the policy cash value as collateral.
  - Interest rate on these loans could either be fixed (pre-specified at policy issue) or variable (use prevailing rate at time loan is taken).

Example 13.3 - on page 449

- Consider Example 13.3 - check out the policy features and assumptions
- Tables in subsequent pages show the emergence of the account value and cash value for 20 years for: $i^c = i^g = 5\%$
  - policyholder pays premium of $2,250 each year for 20 years
  - policyholder pays premium of $2,250 for 6 years, and nothing thereafter

Type B UL ADB is fixed = $100,000
CoI 120% of q of std Select Mortality Model
Expenses are e = $48 - f = 1% of premium

$$AV_1 = \underbrace{(0 + 2250(1-.01) - 48)(1.05) - 1.2 * .0006592 * 100{,}000}_{2209.37}$$

$$AV_2 = \underbrace{(2209.37 + 2250(1-.01) - 48)(1.05) - 1.2 * .0007973 * 100{,}000}_{4512.63}$$

$$CV = \max(AV - SC, \phi)$$

$$\frac{2209.37 + 100{,}006}{2209.37} = 46.3$$

SC:

Year	1	2	3-4	5-7	8-10	710
	4500	4100	3500	2500	1200	ϕ

Detailed results

$AV_{k-1} + \Pi_{k-1} - EC_{k-1} - COI_{k-1} + IC_k$

year k	premium π_{k-1}	expense charge EC_{k-1}	$q_{[45]+k-1}$	COI_{k-1}	interest credited IC_k	account value AV_k	CV_k	corridor factor
1	2250	70.50	0.0006592	75.34	105.21	2209.37	0.00	46.3
2	2250	70.50	0.0007973	91.13	214.89	4512.63	412.63	23.2
3	2250	70.50	0.0009162	104.71	329.37	6916.79	3416.79	15.5
4	2250	70.50	0.0010025	114.57	449.09	9430.80	5930.80	11.6
5	2250	70.50	0.0010995	125.66	574.23	12058.87	9558.87	9.3
6	2250	70.50	0.0012085	138.12	705.01	14805.27	12305.27	7.8
7	2250	70.50	0.0013310	152.12	841.63	17674.28	15174.28	6.7
8	2250	70.50	0.0014687	167.85	984.30	20670.23	19470.23	5.8
9	2250	70.50	0.0016235	185.54	1133.21	23797.40	22597.40	5.2
10	2250	70.50	0.0017974	205.41	1288.57	27060.06	25860.06	4.7
11	2250	70.50	0.0019928	227.75	1450.59	30462.40	30462.40	4.3
12	2250	70.50	0.0022124	252.84	1619.45	34008.51	34008.51	3.9
13	2250	70.50	0.0024592	281.05	1795.35	37702.31	37702.31	3.7
14	2250	70.50	0.0027365	312.74	1978.45	41547.53	41547.53	3.4
15	2250	70.50	0.0030481	348.35	2168.93	45547.61	45547.61	3.2
16	2250	70.50	0.0033982	388.37	2366.94	49705.68	49705.68	3.0
17	2250	70.50	0.0037916	433.33	2572.59	54024.44	54024.44	2.9
18	2250	70.50	0.0042336	483.84	2786.01	58506.11	58506.11	2.7
19	2250	70.50	0.0047302	540.59	3007.25	63152.27	63152.27	2.6
20	2250	70.50	0.0052880	604.34	3236.37	67963.80	67963.80	2.5

Additional details of calculations

- premium π_{k-1} of \$ 2,250 is paid at the beginning of year k
- expense charge $\mathsf{EC}_{k-1} = \pi_{k-1} \times f + e$ where $f = 1\%$ and $e = 48$
- $q_{[45]+k-1}$ is the rate of mortality based on the Standard Select Survival Model
- cost of insurance $\mathsf{COI}_{k-1} = 100,000 \times \dfrac{1}{1+i^q} \times 1.2\, q_{[45]+k-1}$ where $i^q = i^c = 5\%$
- interest credited $\mathsf{IC}_k = [\mathsf{AV}_{k-1} + \pi_{k-1}(1-f) - e - \mathsf{COI}_{k-1}] \times i^c$
- cash value $\mathsf{CV}_k = \max(\mathsf{AV}_k - \mathsf{SC}_k, 0)$
- corridor factor is $\dfrac{\mathsf{AV}_k + \mathsf{ADB}_k}{\mathsf{AV}_k}$

Detailed results - Table 13.4

year k	premium π_{k-1}	expense charge EC_{k-1}	$q_{[45]+k-1}$	COI_{k-1}	interest credited IC_k	account value AV_k	CV_k	corridor factor
1	2250	70.50	0.0006592	75.34	105.21	2209.37	0.00	46.3
2	2250	70.50	0.0007973	91.13	214.89	4512.63	412.63	23.2
3	2250	70.50	0.0009162	104.71	329.37	6916.79	3416.79	15.5
4	2250	70.50	0.0010025	114.57	449.09	9430.80	5930.80	11.6
5	2250	70.50	0.0010995	125.66	574.23	12058.87	9558.87	9.3
6	2250	70.50	0.0012085	138.12	705.01	14805.27	12305.27	7.8
7	0	48.00	0.0013310	152.12	730.26	15335.41	12835.41	7.5
8	0	48.00	0.0014687	167.85	755.98	15875.53	14675.53	7.3
9	0	48.00	0.0016235	185.54	782.10	16424.09	15224.09	7.1
10	0	48.00	0.0017974	205.41	808.53	16979.22	15779.22	6.9
11	0	48.00	0.0019928	227.75	835.17	17538.64	17538.64	6.7
12	0	48.00	0.0022124	252.84	861.89	18099.69	18099.69	6.5
13	0	48.00	0.0024592	281.05	888.53	18659.17	18659.17	6.4
14	0	48.00	0.0027365	312.74	914.92	19213.35	19213.35	6.2
15	0	48.00	0.0030481	348.35	940.85	19757.85	19757.85	6.1
16	0	48.00	0.0033982	388.37	966.07	20287.56	20287.56	5.9
17	0	48.00	0.0037916	433.33	990.31	20796.54	20796.54	5.8
18	0	48.00	0.0042336	483.84	1013.24	21277.94	21277.94	5.7
19	0	48.00	0.0047302	540.59	1034.47	21723.82	21723.82	5.6
20	0	48.00	0.0052880	604.34	1053.57	22125.05	22125.05	5.5

Illustrative example 1

For a Universal Life policy with death benefit equal to \$ 4,500 plus account value issued to (50), you are given:

- The premium paid at the beginning of the first year is \$ 1,000.
- Expense charges in each year are 1.5% of premium plus \$ 20.
- The cost of insurance rate is equal to 125% of the mortality rate at the attained age based on the Illustrative Life Table.
- $i^c = 5\%$ for all years
- $i^q = 4\%$ for all years
- The account value at the end of the second year is equal to \$ 2,238.11.

1. Calculate the premium paid at the beginning of the second year.
2. If the corridor factor requirement is a minimum of 2.5 each year, calculate the largest amount of premium this policyholder can pay at the beginning of the second year.

$AV_0 = 0$

$$AV_1 = \frac{\left[0 + 1000(1-.015) - 20 - 1.25\,{}_{50}\!\left(\frac{1}{1.04}\right)4500\right] \times 1.05}{\text{(5.92/1000)}}$$

$$= 979.6298$$

$$AV_2 = \left[979.6298 + \Pi_1(1-.015) - 20 - 1.25\,{}_{51}\!\left(\frac{1}{1.04}\right)\cdot 4500\right] \times 1.05$$

AV_2 ↙ 2238.11

$1.25\,{}_{51}\!\left(\frac{1}{1.04}\right)$ ↙ 6.42/1000

$$\Pi_1 = \frac{\frac{2238.11}{1.05} - 979.6298 + COI}{1-.015} = 1225.002$$

② $cf = \dfrac{AV_2 + 4500}{AV_2} \geq 2.5 \Rightarrow AV_2 \leq \dfrac{4500}{1.5} = 3000$

$\left(979.6298 + \pi_1(1-.015) - 20 - 1.25\left(\dfrac{6.42}{1000}\right)\dfrac{1}{1.04}4500\right)(1.05) \leq 3000$

Solve for π_1

$\pi_1 \leq \dfrac{\dfrac{3000}{1.05} - 979.6298 + 20 + 1.25\left(\dfrac{6.42}{1000}\right)\left(\dfrac{1}{1.04}\right)4500}{1-.015}$

$= 1,961.662$

no MW class

5/6. 2-hr exam no MC

GUL

12 questions
show work

tables provided
bring calculator
3 ~~sheets~~ formula sheets.

Pricing
Reserves
MS models
- MD
- ML

Review Session
Thu, 5-8pm Room C-304

Monday, 3-5pm May 4

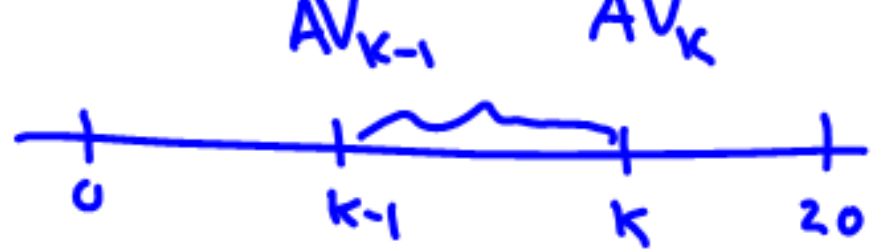
SOA question #297

For a universal life insurance on (50), you are given:

- The death benefit is 100,000.
- Death benefits are paid at the end of the year of death if (50) dies prior to age 70.
- The account value is calculated annually.
- Level annual premiums are payable at the beginning of each year.
- Mortality rates for calculating the cost of insurance follow the Illustrative Life Table.
- Interest is credited at an annual effective rate of 0.06.
- The interest rate used for accumulating and discounting in the cost of insurance calculation is an annual effective rate of 0.06.
- Expense deductions are: 50 at the beginning of each year and 5% of each annual premium.

Calculate the level annual premium that results in an account value of 0 at the end of the 20th year.

Π = premium



$$AV_k = \left[AV_{k-1} + \Pi(1-5\%) - 50\right](1.06) - (100000 - AV_k)\, q_{50+k-1}$$

P

$$AV_k(1 - q_{50+k-1}) = \left[AV_{k-1} + .95\Pi - 50\right](1.06) - 100{,}000\, q_{50+k-1}$$

$$AV_k = \frac{\left[AV_{k-1} + .95\Pi - 50\right](1.06) - 100{,}000\, q_{50+k-1}}{1 - q_{50+k-1}}$$

Recursive reserve formula

$AV_{20} = 0$

$P = \cancel{100000A} \Rightarrow$ term insurance of 100,000 for 20 years to (50)

$$P = \frac{100{,}000\ A^{1}_{50:\overline{20}|} - \underbrace{A_{50} - {}_{20}E_{50}\,A_{70}}_{= 0.13037764}}{\ddot{a}_{50:\overline{20}|}}$$

$\ddot{a}_{50:\overline{20}|} \rightarrow 11.29183$

$\ddot{a}_{50} - {}_{20}E_{50}\,\ddot{a}_{70}$

$= 1154.608$

Thus, $P = \underbrace{.95\,\Pi - 50 = 1154.608}$

$$\Rightarrow \quad \Pi = \frac{1154.608 - 50}{.95} = \underline{\underline{1268.008}}$$

SOA question #11, Spring 2012

type B

For a universal life insurance policy with death benefit of 10,000 plus account value, you are given:

Policy Year	Monthly Premium	Percent of Premium Charge	Cost of Insurance Rate Per Month	Monthly Expense Charge	Surrender Charge
1	100	30%	0.001	5	300
2	100	10%	0.002	5	100

- The credited interest rate is $i^{(12)} = 0.048$. $\Rightarrow \frac{.048}{12} = .004$
- The actual cash surrender value at the end of month 11 is 1000.
- The policy remains in force for months 12 and 13, with the monthly premiums of 100 being paid at the start of each month.

Calculate the cash surrender value at the end of month 13.

$\hookrightarrow = AV_{13} - SC$

$AV_{11} = CV + SC = 1000 + 300 = 1300$

months

$AV_{12} = \left(AV_{11} + 100(1-.3) - 5\right)(1.004) - \left(10{,}000 + AV_{12} - AV_{12}\right) * .001$

$= 1360.46$

$AV_{13} = \left(AV_{12} + 100(1-.1) - 5\right)(1.004) - 10{,}000\,(.002)$

1431.242

100

Surrender Value $= 1431.242 - SC = 1{,}331.242$

SOA question #9, Fall 2012

You are given the following about a universal life insurance policy on (60):

Age x	Annual Premium	Annual Cost of Insurance Rate per 1000	Annual Expense Charges
60	5000	5.40	100
61	5000	6.00	100

- The death benefit equals the account value plus 200,000. type B
- Interest is credited at 6%.
- Surrender value equals 93% of account value during the first two years. Surrenders occur at the ends of policy years.
- Surrenders are 6% per year of those who survive.
- Mortality rates are $1000\,q_{60} = 3.40$ and $1000\,q_{61} = 3.80$.
- $i = 7\%$

Calculate the present value at issue of the insurer's expected surrender benefits paid in the second year.

$AV_0 = 0$

$AV_1 = \underbrace{(5000 - 100)(1.06) - 200{,}000 * \frac{5.4}{1000}}_{4{,}114}$

$AV_2 = \underbrace{(AV_1 + 5000 - 100)(1.06) - 200{,}000 * \frac{6.0}{1000}}_{8354.84}$

Surrender Benefit $= .93(8354.84) =$

Expected Surrender Benefit at time 2 values at time 0 $= .93(8354.84) * \left(1 - \frac{3.4}{1000}\right)\left(1 - \frac{3.8}{1000}\right) * .06 * .94 * \frac{1}{1.07^2}$

$= 380.0144$

SOA question #18, Fall 2014

For a Type B universal life insurance policy, you are given:

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Additional Death Benefit	Annual Cost of Insurance (COI) Rate	Anuual Discount Rate for COI	Annual Credited Interest Rate
1	2500	1%	50	100,000	0.0028	5.0%	4.5%
2	3000	1%	50	95,000	0.0030	5.0%	5.2%

Calculate the account value at the end of year 2.

$AV_0 = 0$

$$AV_1 = \left(2500(1-.01) - 50 - 100{,}000 * .0028 * \frac{1}{1.05}\right)(1.045)$$

$$= 2255.45$$

$$AV_2 = \left(2255.45 + 3000(1-.01) - 50 - 95000 * .0030 * \frac{1}{1.05}\right)(1.052)$$

$$= 5159.03$$

Fall 2013 Q#23 — $i = 4\%$

Type B — $150{,}000 + AV$ to (25)

Year	Annual Prem	% of Prem	COI per 1000	Annual expenses	AV_{end}
			1.22	75	–
1	3000	70%	1.27	R	–
2	3000	10%	1.33	R	6028.95
3	3000	10%			

$AV_0 = 0$

$$AV_1 = \frac{(3000(1-.7) - 75)(1.04) - 150000 * \frac{1.22}{1000}}{675}$$

$$AV_2 = (675 + 3000(1-.1) - R)(1.04) - 150{,}000 * \frac{1.27}{1000}$$

$$= 3319.5 - 1.04R$$

$$AV_3 = (3319.5 - 1.04R + 3000(1-.1) - R)(1.04)$$

$$- 150{,}000 * 1.3\%_{1000}$$

$$= \underbrace{6060.78 - R(1.04^2 + 1.04)}_{= 6028.95}$$

$$R = \frac{6060.78 - 6028.95}{1.04^2 + 1.04} = 15.00283$$

- x Type A / Type B
- * corridor factor

10-132 Pension math