

# Premium Calculation - continued

Lecture: Weeks 1-2

## Recall some preliminaries

An insurance policy (life insurance or life annuity) is funded by contract premiums:

- once (single premium) made usually at time of policy issue, or
- a series of payments (usually contingent on survival of policyholder) with first payment made at policy issue
- to cover for the benefits, expenses associated with initiating/maintaining contract, profit margins, and deviations due to adverse experience.

**Net premiums** (or sometimes called **benefit premiums**)

- considers only the benefits provided
- nothing allocated to pay for expenses, profit or contingency margins

**Gross premiums** (or sometimes called **expense-loaded premiums**)

- covers the benefits and includes expenses, profits, and contingency margins



## Chapter summary - continued

- Present value of future loss (negative profit) random variable
- Premium principles
  - the **equivalence principle** (or actuarial equivalence principle)
  - portfolio percentile premiums
- Return of premium policies
- Substandard risks
- Chapter 6 (sections 6.8, 6.9) of Dickson, et al.

## Net random future loss

- An insurance contract is an agreement between two parties:
  - the insurer agrees to pay for insurance benefits;
  - in exchange for insurance premiums to be paid by the insured.
- The insurer's **net random future loss** is defined by

$$L_0^n = PVFB_0 - PVFP_0.$$

where  $PVFB_0$  is the present value, at time of issue, of future benefits to be paid by the insurer

and

$PVFP_0$  is the present value, at time of issue, of future premiums to be paid by the insured.

- Note: this is also called the present value of future loss random variable (in the book), and if no confusion, we may simply write this as  $L_0$ .



## The principle of equivalence

- The **net premium**, generically denoted by  $P$ , may be determined according to the **principle of equivalence** by setting

$$E[L_0^n] = 0.$$

- The expected value of the insurer's net random future loss is zero.
- This is then equivalent to setting  $E[PVFB_0] = E[PVFP_0]$ . In other words, at issue, we have

$$APV(\text{Future Premiums}) = APV(\text{Future Benefits}).$$

## Gross premium calculations

- Treat expenses as if they are a part of benefits. The **gross random future loss** at issue is defined by

$$L_0^g = PVFB_0 + PVFE_0 - PVFP_0,$$

where  $PVFE_0$  is the present value random variable associated with future expenses incurred by the insurer.

- The **gross premium**, generically denoted by  $G$ , may be determined according to the **principle of equivalence** by setting

$$E[L_0^g] = 0.$$

- This is equivalent to setting  $E[PVFB_0] + E[PVFE_0] = E[PVFP_0]$ . In other words, at issue, we have

$$APV(FP_0) = APV(FB_0) + APV(FE_0).$$



## Portfolio percentile premium principle

Suppose insurer issues a portfolio of  $N$  “identical” and “independent” policies where the PV of loss-at-issue for the  $i$ -th policy is  $L_{0,i}$ .

The total portfolio (aggregate) future loss is then defined by

$$L_{\text{agg}} = L_{0,1} + L_{0,2} + \cdots + L_{0,N} = \sum_{i=1}^N L_{0,i}$$

Its expected value is therefore

$$E[L_{\text{agg}}] = \sum_{i=1}^N E[L_{0,i}]$$

and, by “independence”, the variance is

$$\text{Var}[L_{\text{agg}}] = \sum_{i=1}^N \text{Var}[L_{0,i}].$$

## Portfolio percentile premium principle

The **portfolio percentile premium principle** sets the premium  $P$  so that there is a probability, say  $\alpha$  with  $0 < \alpha < 1$ , of a positive gain from the portfolio.

In other words, we set  $P$  so that

$$\Pr[L_{\text{agg}} < 0] = \alpha.$$

Note that loss could include expenses.



## Example 6.12

Consider Example 6.12.

## Illustrative example 1

An insurer sells 100 fully discrete whole life insurance policies of \$1, each of the same age 45. You are given:

- All policies have independent future lifetimes.
- $i = 5\%$
- $\ddot{a}_{45} = 17.81876$

Using the Normal approximation:

- 1 Calculate the annual contract premium according to the portfolio percentile premium principle with  $\alpha = 0.95$ .
- 2 Suppose the annual contract premium is set at 0.01 per policy. Determine the smallest number of policies to be sold so that the insurer has at least a 95% probability of a gain from this portfolio of policies.



## Return of premium policies

Consider a fully discrete whole life insurance to  $(x)$  with benefit equal to  $\$B$  plus **return of all premiums** accumulated with interest at rate  $j$ .

The net random future loss in this case can be expressed as

$$L_0 = P \ddot{s}_{\overline{K+1}|j} v^{K+1} + B v^{K+1} - P \ddot{a}_{\overline{K+1}|},$$

for  $K = 0, 1, \dots$  and  $\ddot{s}_{\overline{K+1}|j}$  is calculated at rate  $j$ . All other actuarial functions are calculated at rate  $i$ .

Consider the following cases:

- Let  $j = 0$ . This implies  $\ddot{s}_{\overline{K+1}|j} = (K + 1)$  and the annual benefit premium will be

$$P = \frac{B A_x}{\ddot{a}_x - (IA)_x}.$$

## - continued

- Let  $i = j$ . In this case, the loss  $L_0 = B v^{K+1}$  because  $\ddot{s}_{\overline{K+1}|j} v^{K+1} = \ddot{a}_{\overline{K+1}|}$ . Thus, there is no possible premium because all premiums are returned and yet there is an additional benefit of  $\$B$ .
- Let  $i < j$ . Then we have

$$L_0 = P \left( \ddot{s}_{\overline{K+1}|j} - \ddot{s}_{\overline{K+1}|} \right) v^{K+1} + B v^{K+1},$$

which is always positive because  $\ddot{s}_{\overline{K+1}|j} > \ddot{s}_{\overline{K+1}|}$  when  $i < j$ . No possible premium.

## - continued

- Let  $i > j$ . Then we can write the loss as

$$L_0 = P \frac{v_{j^*}^{K+1} - v^{K+1}}{d_j} + B v^{K+1} - P \ddot{a}_{\overline{K+1}|}$$

where  $d_j = 1 - [1/(1 + j)]$  and  $v_{j^*}$  is the corresponding discount rate associated with interest rate  $j^* = [(1 + i)/(1 + j)] - 1$ . Here,

$$P = \frac{A_x}{\ddot{a}_x - \frac{(A_x)_{j^*} - A_x}{d_j}},$$

where  $(A_x)_{j^*}$  is a (discrete) whole life insurance to  $(x)$  evaluated at interest rate  $j^*$ .



## Illustrative example 2

For a whole life insurance on  $(40)$ , you are given:

- Death benefit, payable at the end of the year of death, is equal to \$10,000 plus the return of all premiums paid without interest.
- Annual benefit premium of 290.84 is payable at the beginning of each year.
- $(IA)_{40} = 8.6179$
- $i = 4\%$

Calculate  $\ddot{a}_{40}$ .

## SOA Question #22

Consider Question #22 from Fall 2012 SOA MLC Exam:

You are given the following information about a special fully discrete 2-payment, 2-year term insurance on (80):

- Mortality follows the Illustrative Life Table.
- $i = 0.0175$
- The death benefit is 1000 plus a return of all premiums paid without interest.
- Level premiums are calculated using the equivalence principle.

Calculate the benefit premium for this special insurance.

For practice: try calculating the benefit premium if the return of all premiums paid comes with an interest of say 0.01.

## Pricing with extra or substandard risks

An impaired individual, or one who suffers from a medical condition, may still be offered an insurance policy but at a rate higher than that of a standard risk.

Generally there are three possible approaches:

- **age rating**: calculate the premium with the individual at an older age
- **constant addition to the force of mortality**:  $\mu_{x+t}^s = \mu_{x+t} + \phi$ , for  $\phi > 0$
- **constant multiple of mortality rates**:  $q_{x+t}^s = \min(cq_{x+t}, 1)$ , for  $c > 1$

Read Section 6.9.





## Published SOA question #45

Your company is competing to sell a life annuity-due with an APV of \$500,000 to a 50-year-old individual.

Based on your company's experience, typical 50-year-old annuitants have a complete life expectancy of 25 years. However, this individual is not as healthy as your company's typical annuitant, and your medical experts estimate that his complete life expectancy is only 15 years.

You decide to price the benefit using the issue age that produces a complete life expectancy of 15 years. You also assume:

- For typical annuitants of all ages,  $l_x = 100(\omega - x)$ , for  $0 \leq x \leq \omega$ .
- $i = 0.06$

Calculate the annual benefit premium that your company can offer to this individual.



## Other terminologies and notations used

Expression	Other terms/symbols used
net random future loss	loss at issue
$L_0$	${}_0L$
net premium	benefit premium
gross premium	expense-loaded premium
equivalence principle	actuarial equivalence principle
generic premium	$G$ $P$ $\pi$
substandard	may be superscripted with * or $s$