Exercise 7.1

(a) Let P be the required annual benefit premium and by the equivalence principle, we have

$$P = 200000 \times \frac{A^{\;1}_{[41]:\overline{3}|}}{\ddot{a}_{[41]:\overline{3}|}},$$

where

$$\ddot{a}_{[41]:\overline{3}|} = 1 + vp_{[41]} + v^2{}_2p_{[41]} = 1 + \frac{1}{1.06}\frac{99689}{99802} + \frac{1}{1.06^2}\frac{99502}{99802} = 2.829644$$

and

$$\begin{aligned} A^{\;1}_{[41]:\overline{3}|} &= A_{[41]:\overline{3}|} - {}_3E_{[41]} = \left(1 - d\ddot{a}_{[41]:\overline{3}|}\right) - v^3{}_3p_{[41]} \\ &= \left[1 - (1 - (1.06)^{-1})(2.829644)\right] - \frac{1}{1.06^3}\frac{99283}{99802} = 0.004578162. \end{aligned}$$

Thus, it follows that

$$P = 200000 \times \frac{0.004578162}{2.829644} = 323.5851.$$

(b) Simply denote $K_{[41]+1}$ by K. We have

$$L_1 = \text{PVFB}_1 - \text{PVFP}_1 = \begin{cases} 200000v^{K+1} - P\ddot{a}_{\overline{K+1}|}, & \text{for } K < 2 \\ -P\ddot{a}_{\overline{2}|}, & \text{for } K \geq 2 \end{cases}$$

The following table provides details of the calculations for the expected value and standard deviation of L_1:

k	$\Pr[K = k]$	$L_1 = \ell$	$\ell \cdot \Pr[K = k]$	$\ell^2 \cdot \Pr[K = k]$
0	0.001875834	$200000v - P = 188355.6602$	353.3239	66550560.6
1	0.002196832	$200000v^2 - P(1 + v) = 177370.4339$	389.6531	69112934.3
≥ 2	0.995927334	$-P(1 + v) = -628.8541$	-626.2930	393846.9
sum	1.00000		116.6840	136057342

Thus, we find from this table that

$$\text{E}[L_1] = 116.6840 \quad \text{and} \quad \text{E}[L_1^2] = 136057342$$

so that the required standard deviation is given by

$$\text{SD}[L_1] = \sqrt{\text{E}[L_1^2] - (\text{E}[L_1])^2} = \sqrt{136057342 - (116.6840)^2} = 11663.78.$$

(c) Let B be the required sum insured so that B satisfies

$$P \times \ddot{a}_{[41]:\overline{3}|} = B \times A_{[41]:\overline{3}|}.$$

The solution is therefore

$$\begin{aligned} B &= P \times \frac{\ddot{a}_{[41]:\overline{3}|}}{A_{[41]:\overline{3}|}} = P \times \frac{\ddot{a}_{[41]:\overline{3}|}}{1-(1-v)\ddot{a}_{[41]:\overline{3}|}} \\ &= 323.5851 \times \frac{2.829644}{1-(1-(1/1.06))2.829644} = 1090.258. \end{aligned}$$

(d) Following the procedure in (b), we provide the table below for the details of the calculations for the expected value and standard deviation of the corresponding L_1:

k	$\Pr[K=k]$	$L_1 = \ell$	$\ell \cdot \Pr[K=k]$	$\ell^2 \cdot \Pr[K=k]$
0	0.001875834	$Bv - P = 704.9599$	1.322388	932.2302
≥ 1	0.998124166	$Bv^2 - P(1+v) = 341.4714$	340.830815	116383.9616
sum	1.00000		342.1532	117316.2

Thus, we find from this table that

$$\mathrm{E}[L_1] = 342.1532 \quad \text{and} \quad \mathrm{E}[L_1^2] = 117316.2$$

so that the required standard deviation is given by

$$\mathrm{SD}[L_1] = \sqrt{\mathrm{E}[L_1^2] - (\mathrm{E}[L_1])^2} = \sqrt{117316.2 - (342.1532)^2} = 15.72824.$$

(e) Because of the extra payment of the pure endowment in an endowment policy, this leads to a larger expected future loss, but smaller variation than that of a term insurance.