## Exercise 7.10

For the $n$-year endowment policy as described in this problem, the net premium policy value at duration $t$, provided $t \leq n$, can be expressed as

$$\begin{aligned} {}_tV &= \text{APV}(\text{FB}_t) - \text{APV}(\text{FP}_t) \\ &= S \times A^{(12)}_{[x]+t:\overline{n-t}|} - P \times \ddot{a}^{(12)}_{[x]+t:\overline{n-t}|}, \end{aligned}$$

where the net premium, determine according to the equivalence principle can be expressed as

$$P = S \times \frac{A^{(12)}_{[x]:\overline{n}|}}{\ddot{a}^{(12)}_{[x]:\overline{n}|}} = S \times \frac{1 - d^{(12)}\ddot{a}^{(12)}_{[x]:\overline{n}|}}{\ddot{a}^{(12)}_{[x]:\overline{n}|}} = S \times \left( \frac{1}{\ddot{a}^{(12)}_{[x]:\overline{n}|}} - d^{(12)} \right).$$

Substituting this back to the policy value formula, we get

$$\begin{aligned} {}_tV &= S \times A^{(12)}_{[x]+t:\overline{n-t}|} - S \times \left( \frac{1}{\ddot{a}^{(12)}_{[x]:\overline{n}|}} - d^{(12)} \right) \times \ddot{a}^{(12)}_{[x]+t:\overline{n-t}|} \\ &= S \times \left[ \left( A^{(12)}_{[x]+t:\overline{n-t}|} + d^{(12)}\ddot{a}^{(12)}_{[x]+t:\overline{n-t}|} \right) - \frac{\ddot{a}^{(12)}_{[x]+t:\overline{n-t}|}}{\ddot{a}^{(12)}_{[x]:\overline{n}|}} \right] \\ &= S \times \left[ 1 - \frac{\ddot{a}^{(12)}_{[x]+t:\overline{n-t}|}}{\ddot{a}^{(12)}_{[x]:\overline{n}|}} \right], \end{aligned}$$

since clearly we have

$$A^{(12)}_{[x]+t:\overline{n-t}|} + d^{(12)}\ddot{a}^{(12)}_{[x]+t:\overline{n-t}|} = 1$$

which therefore proves the desired result.