Exercise 7.12

(a) Let G be the required annual gross premium. The APV of future premiums is

$$\text{APV}(\text{FP}_0) = G\ddot{a}_{[40]:\overline{10}|},$$

the APV of future benefits is

$$\text{APV}(\text{FB}_0) = 20000 A_{[40]:\overline{10}|} - 10000\,{}_{10}E_{[40]},$$

and the APV of future expenses is

$$\text{APV}(\text{FE}_0) = 0.05 \times G\ddot{a}_{[40]:\overline{10}|}.$$

By the equivalence principle, we have the gross annual premium equal to

$$G = \frac{10000\left(2A_{[40]:\overline{10}|} - {}_{10}E_{[40]}\right)}{0.95\ddot{a}_{[40]:\overline{10}|}}.$$

Based on the Standard Select Survival Model with $i = 5\%$, one can verify that

$$\ddot{a}_{[40]:\overline{10}|} = \ddot{a}_{[40]} - {}_{10}E_{[40]}\ddot{a}_{50} = 18.45956 - (0.6092688)17.02453 = 8.087046$$

and that

$$A_{[40]:\overline{10}|} = 1 - (.045/1.045)(8.087046) = 0.6149026$$

so that we have

$$G = \frac{10000\,[2(0.6149026) - 0.6092688]}{0.95(8.087046)} = 807.7068.$$

(b) The policy value at each year end can be evaluated recursively. Starting with ${}_0V = 0$, we have

$${}_kV = \frac{({}_{k-1}V + 0.95G)(1+i) - 20000q_{[40]+k-1}}{1 - q_{[40]+k-1}},$$

for $k = 1, 2, \ldots, 9$. And for $k = 10$, we have

$${}_{10}V = \frac{({}_9V + 0.95G)(1+i) - 20000q_{49} - 10000(1-q_{49})}{1 - q_{49}},$$

because of the endowment of 10000 if the life survives to reach age 50. The following calculations can easily be verified:

$${}_1V = \frac{({}_0V + 0.95G)(1.05) - 10000q_{[40]}}{1 - q_{[40]}} = 797.0338,$$

$${}_2V = \frac{({}_1V + 0.95G)(1.05) - 10000q_{[40]+1}}{1 - q_{[40]+1}} = 1632.7117,$$

and so forth, until we have

$${}_4V = \frac{({}_3V + 0.95G)(1.05) - 10000q_{43}}{1 - q_{43}} = 3429.6815,$$

the required policy value just prior to the payment of the fifth premium. One can also verify, as expected, that ${}_{10}V = 0$.

(c) Let B denote the revised death benefit. We solve for B based on the following equation of value with the requested changes:

$$ {}_4V + \text{APV}(\text{FP}_4) = \text{APV}(\text{FB}_4) + \text{APV}(\text{FE}_4), $$

where

$$ \text{APV}(\text{FP}_4) = \frac{1}{2} G \ddot{a}_{44:\overline{6}|}, $$

$$ \text{APV}(\text{FB}_4) = B A_{44:\overline{6}|} - \frac{1}{2} B \, {}_6E_{44}, $$

and

$$ \text{APV}(\text{FE}_4) = 0.05 \times \frac{1}{2} G \ddot{a}_{44:\overline{6}|}. $$

Solving for the revised death benefit, we arrive at

$$ B = \frac{{}_4V + 0.95\frac{1}{2} G \ddot{a}_{44:\overline{6}|}}{A_{44:\overline{6}|} - \frac{1}{2} \, {}_6E_{44}}. $$

Substituting the values, we have

$$ B = \frac{3429.682 + 0.95\frac{1}{2}(807.7068)(5.319477)}{0.7466916 - \frac{1}{2}(0.74224)} = 14565.95. $$