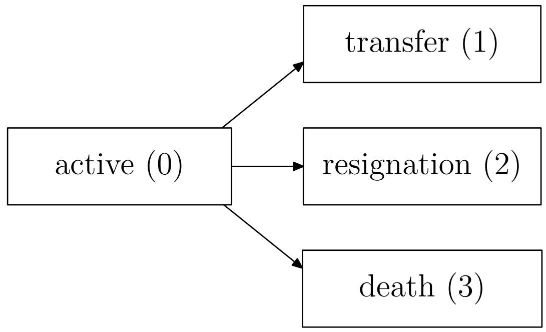
Exercise 8.12

This problem is best depicted by the figure below:



First, we note that given the transition intensities, we can find an explicit expression for the probability of a recruit age x staying active for t years as

$$\begin{aligned} {}_tp_x^{00} &= \exp\left[-\int_0^t \left(\mu_{x+s}^{01} + \mu_{x+s}^{02} + \mu_{x+s}^{03}\right) ds\right] \\ &= \mathrm{e}^{-\frac{0.001}{2}[(x+t)^2 - x^2]} \times \mathrm{e}^{-0.01t} \times \mathrm{e}^{-\left[At + \frac{Bc^x}{\log(c)}(c^t - 1)\right]} \end{aligned}$$

(a) A new recruit is age 25.

(i) The probability a new recruit is transfered before reaching age 27 is

$$\int_0^2 {}_tp_{25}^{00}\, \mu_{25+t}^{01} dt = 0.05000176.$$

```
x <- 25
A <- 0.001
B <- 0.0004
c <- 1.07
mux01 <- function(t){
out <- 0.001*(x+t)
out}
mux02 <- 0.01
mux03 <- function(t){
out <- A + B*c^(x+t)
out}
tpx00 <- function(t){
temp1 <- (0.001/2)*((x+t)^2 - x^2)
temp2 <- 0.01*t
temp3 <- A*t + B*c^x*(c^t - 1)/log(c)
out <- exp(-temp1-temp2-temp3)
out}
h <- 1/1000
t <- seq(0,2,h)
intai <- tpx00(t)*mux01(t)
probai <- 0
```

```
n <- 1
while (n<length(t)) {
n <- n+2
probai <- probai + (h/3)*(intai[n-2]+4*intai[n-1]+intai[n])
}

> probai
[1] 0.05000176
```

(ii) For the probability that a new recruit dies aged 27 last birthday, he must survive the first two years from recruitment and then dies the following year:

$$ {}_2p_{25}^{00} \times \int_0^1 {}_tp_{27}^{00}\,\mu_{27+t}^{03} dt = 0.00323432. $$

(iii) The probability that a new recruit remain in active service at age 28 is

$$ {}_3p_{25}^{00} = 0.887168. $$

```
factor1 <- tpx00(2)
factoraiii <- tpx00(3)
x <- 27
mux01 <- function(t){
out <- 0.001*(x+t)
out}
mux02 <- 0.01
mux03 <- function(t){
out <- A + B*c^(x+t)
out}
tpx00 <- function(t){
temp1 <- (0.001/2)*((x+t)^2 - x^2)
temp2 <- 0.01*t
temp3 <- A*t + B*c^x*(c^t - 1)/log(c)
out <- exp(-temp1-temp2-temp3)
out}
t <- seq(0,1,h)
intaii <- tpx00(t)*mux03(t)
factor2 <- 0
n <- 1
while (n<length(t)) {
n <- n+2
factor2 <- factor2 + (h/3)*(intaii[n-2]+4*intaii[n-1]+intaii[n])
}
probaii <- factor1*factor2
probaiii <- factoraiii

> probaii
[1] 0.00323432
```

```
> probaiii
[1] 0.887168
```

(b) Let P be the levy at each anniversary in the first two years from recruitment. The APV of future levy will be

$$\text{APV(levy)} = P \times \left(v \, {}_1p_{25}^{00} + v^2 \, {}_2p_{25}^{00} \right) = P \times 1.730223.$$

The APV of the lump sum payment payable immediately upon transfer is given by

$$\text{APV(transfer benefit)} = 10000 \times \int_0^3 v^t \, {}_tp_{25}^{00} \, \mu_{25+t}^{01} dt = 687.3086.$$

Solving for P, we have

$$P = \frac{687.3086}{1.730223} = 397.237.$$

```
x <- 25
A <- 0.001
B <- 0.0004
c <- 1.07
mux01 <- function(t){
out <- 0.001*(x+t)
out}
mux02 <- 0.01
mux03 <- function(t){
out <- A + B*c^(x+t)
out}
tpx00 <- function(t){
temp1 <- (0.001/2)*((x+t)^2 - x^2)
temp2 <- 0.01*t
temp3 <- A*t + B*c^x*(c^t - 1)/log(c)
out <- exp(-temp1-temp2-temp3)
out}
h <- 1/1000
t <- seq(0,3,h)
i<- 0.06
v <- 1/(1+i)
vt <- v^t
intb <- 10000*vt*tpx00(t)*mux01(t)
apvtb <- 0
n <- 1
while (n<length(t)) {
n <- n+2
apvtb <- apvtb + (h/3)*(intb[n-2]+4*intb[n-1]+intb[n])
}
apvlevy <- v*tpx00(1) + v^2*tpx00(2)
P <- apvtb/apvlevy
```

```
> apvtb
[1] 687.3086
> apvlevy
[1] 1.730223
> P
[1] 397.237
```

(c) Consider the time interval between the age at recruit 25 and age 28. For this new recruit to die before age 28 as an elite force, then first he must transition from recruit to elite force (i.e. transfer) then die before reaching age 28. The required probability then is equal to

$$\int_0^3 {}_sp_{25}^{00}\, \mu_{25+s}^{01} \times {}_{3-s}p_{25+s}^{*13} ds,$$

where the $*$ is to indicate that the corresponding force of mortality upon reaching elite force is different from the original multiple decrement model. Indeed because this new force of mortality is equal to $1.5\mu_x^{03}$, this probability must equal then to

$${}_{3-s}p_{25+s}^{*13} = 1 - \exp\left(-1.5\int_0^t \mu_{x+s}^{03} ds\right) = 1 - \mathrm{e}^{-1.5\left[At + \frac{Bc^x}{\log(c)}(c^t - 1)\right]}.$$

The R code that follows show that the required probability is equal to

$$\int_0^3 {}_sp_{25}^{00}\, \mu_{25+s}^{01} \times {}_{3-s}p_{25+s}^{*13} ds = 0.000585513.$$

```
tpx03 <- function(x,t){
temp <- A*t + B*c^x*(c^t - 1)/log(c)
out <- exp(-1.5*temp)
out}
h <- 1/1000
t <- seq(0,3,h)
intc <- tpx00(t)*mux01(t)*(1 - tpx03(25+t,3-t))
probc <- 0
n <- 1
while (n<length(t)) {
n <- n+2
probc <- probc + (h/3)*(intc[n-2]+4*intc[n-1]+intc[n])
}

> probc
[1] 0.000585513
```