Exercise 8.15

(a) $_3p_{60}^{01} = \dfrac{d_{60}^{(1)} + d_{61}^{(1)} + d_{62}^{(1)}}{l_{60}} = \dfrac{350 + 360 + 380}{10000} = 0.109$

(b) $_2p_{61}^{00} = \dfrac{l_{63}}{l_{61}} = \dfrac{8945 - (380 + 110 + 70)}{9475} = 0.8849604$

(c) The APV of the benefit is

$$10000 \times \left[v \left(\frac{25}{10000} \right) + v^2 \left(\frac{45}{10000} \right) + v^3 \left(\frac{70}{10000} \right) \right] = 125.0945$$

(d) The APV of the annuity benefit is

$$1000 \times \left[1 + v \frac{9475}{10000} + v^2 \frac{8945}{10000} \right] = 2,713.719$$

Note that the answer here does not match exactly that in the book. You should be able to easily verify which one is correct.

(e) Starting with

$$p_{62}^{(\tau)} = e^{-\mu_{62}^{(\tau)}} = \frac{8384}{8945}$$

which implies that

$$\mu_{62}^{(\tau)} = -\log(8384/8945) = 0.06476959$$

Because

$$p_{62}^{01} = \frac{d_{62}^{(1)}}{l_{62}} = \int_0^1 {}_tp_{62}^{(\tau)} \mu_{62+t}^{01} dt = \frac{\mu_{62}^{01}}{\mu_{62}^{(\tau)}} \int_0^1 {}_tp_{62}^{(\tau)} \mu_{62+t}^{(\tau)} dt = \frac{\mu_{62}^{01}}{\mu_{62}^{(\tau)}} \left(1 - p_{62}^{(\tau)} \right),$$

this implies that

$$\mu_{62}^{01} = \frac{d_{62}^{(1)}}{l_{62}} \frac{\mu_{62}^{(\tau)}}{\left(1 - p_{62}^{(\tau)} \right)} = \frac{380}{8945} \frac{0.06476959}{1 - (8385/8945)} = 0.04387245$$

Solving for the required value:

$$q_{62}^{\prime(1)} = 1 - p_{62}^{\prime 01} = 1 - \exp(-\mu_{62}^{01}) = 1 - \exp(-0.04387245) = 0.04292397.$$

(f) With the new $q_{62}^{\prime(1),\text{new}} = 0.1$, this implies the new force of mortality is

$$\mu_{62}^{01,\text{new}} = -\log(1 - 0.1) = 0.1053605$$

(i) Assume constant force assumption: following the method in part (e), we get the forces of decrements

$$\mu_{62}^{02} = \frac{d_{62}^{(2)}}{l_{62}} \frac{\mu_{62}^{(\tau)}}{\left(1 - p_{62}^{(\tau)}\right)} = \frac{110}{8945} \frac{0.06476959}{1 - (8385/8945)} = 0.01269992$$

and

$$\mu_{62}^{03} = \frac{d_{62}^{(3)}}{l_{62}} \frac{\mu_{62}^{(\tau)}}{\left(1 - p_{62}^{(\tau)}\right)} = \frac{70}{8945} \frac{0.06476959}{1 - (8385/8945)} = 0.008081767$$

Because we know that

$$p_{62}^{(\tau),\text{new}} = \exp(-\mu_{62}^{01,\text{new}}) \exp(-\mu_{62}^{02}) \exp(-\mu_{62}^{03}) = 0.8814929$$

so that

$$\ell_{62}^{\text{new}} = \ell_{62} \times p_{62}^{(\tau),\text{new}} = 8945 \times 0.8814929 = 7884.954$$

Finally, we have

$$d_{62}^{(1),\text{new}} = \ell_{62} \times \frac{\mu_{62}^{01,\text{new}}}{\mu_{62}^{(\tau),\text{new}}} \times \left(1 - p_{62}^{(\tau),\text{new}}\right) = 8945 \times \frac{0.01269992}{0.1261422} \times (1 - 0.8814929) = 885.4058$$

and

$$d_{62}^{(2),\text{new}} = \ell_{62} \times \frac{\mu_{62}^{02}}{\mu_{62}^{(\tau),\text{new}}} \times \left(1 - p_{62}^{(\tau),\text{new}}\right) = 8945 \times \frac{0.1053605}{0.1261422} \times (1 - 0.8814929) = 106.7248$$

and

$$d_{62}^{(3),\text{new}} = \ell_{62} \times \frac{\mu_{62}^{03}}{\mu_{62}^{(\tau),\text{new}}} \times \left(1 - p_{62}^{(\tau),\text{new}}\right) = 8945 \times \frac{0.008081767}{0.1261422} \times (1 - 0.8814929) = 67.9158$$

where we note that

$$\mu_{62}^{(\tau),\text{new}} = \mu_{62}^{01,\text{new}} + \mu_{62}^{02} + \mu_{62}^{03} = 0.1053605 + 0.01269992 + 0.008081767 = 0.1261422$$

(ii) Assuming UDD in the single decrement models: first, we note

$$q_{62}^{\prime(1),\text{new}} = 0.1$$
$$q_{62}^{\prime(2)} = 1 - e^{-0.01269992} = 0.01261962$$
$$q_{62}^{\prime(3)} = 1 - e^{-0.008081767} = 0.008049197$$

Thus, we have

$$\begin{aligned} p_{62}^{01,\text{new}} &= \int_0^1 {}_tp_{62}^{(\tau)} \mu_{62+t}^{\prime 01,\text{new}} dt \\ &= q_{62}^{\prime(1)} \int_0^1 \left(1 - tq_{62}^{\prime(2)}\right) \left(1 - tq_{62}^{\prime(3)}\right) dt \\ &= q_{62}^{\prime(1)} \times \left[1 - \frac{1}{2}\left(q_{62}^{\prime(2)} + q_{62}^{\prime(3)}\right) + \frac{1}{3} q_{62}^{\prime(2)} q_{62}^{\prime(3)}\right] \\ &= 0.09896995 \end{aligned}$$

so that

$$d_{62}^{(1),\text{new}} = 8945 \times 0.09896995 = 885.2862$$

Similarly for the other decrements, we have

$$p_{62}^{02,\text{new}} = q_{62}^{\prime(2)} \times \left[1 - \frac{1}{2}\left(q_{62}^{\prime(1),\text{new}} + q_{62}^{\prime(3)}\right) + \frac{1}{3} q_{62}^{\prime(1),\text{new}} q_{62}^{\prime(3)}\right] = 0.01194123$$

so that

$$d_{62}^{(2),\text{new}} = 8945 \times 0.01194123 = 106.8143$$

and finally

$$p_{62}^{03,\text{new}} = q_{62}^{\prime(3)} \times \left[1 - \frac{1}{2}\left(q_{62}^{\prime(1),\text{new}} + q_{62}^{\prime(2)}\right) + \frac{1}{3} q_{62}^{\prime(1),\text{new}} q_{62}^{\prime(2)}\right] = 0.007599334$$

so that

$$d_{62}^{(2),\text{new}} = 8945 \times 0.007599334 = 67.97605.$$