Exercise 8.16

(a) We can find from the Standard Ultimate Survival Model that

$$q_{39}^{'(1)} = 0.00049, \quad q_{40}^{'(1)} = 0.00053, \quad q_{41}^{'(1)} = 0.00057.$$

We need to convert the independent rates to multiple decrement rates. Because resignation occurs only at exact age 40, it only affects the multiple decrement table at that point. According to the assumption of UDD for death and transfers, we have, for deaths:

$$q_{39}^{(1)} = q_{39}^{'(1)}(1 - 0.5q_{39}^{'(2)}) = 0.00049(1 - .5 * .09) = 0.00046795$$

$$q_{40}^{(1)} = q_{40}^{'(1)}(1 - 0.5q_{40}^{'(2)}) = 0.00053(1 - .5 * .10) = 0.0005035$$

$$q_{41}^{(1)} = q_{41}^{'(1)}(1 - 0.5q_{41}^{'(2)}) = 0.00057(1 - .5 * .11) = 0.00053865$$

and for transfers:

$$q_{39}^{(2)} = q_{39}^{'(2)}(1 - 0.5q_{39}^{'(1)}) = 0.09(1 - .5 * 0.00049) = 0.08997795$$

$$q_{40}^{(2)} = q_{40}^{'(2)}(1 - 0.5q_{40}^{'(1)}) = 0.10(1 - .5 * 0.00053) = 0.0999735$$

$$q_{41}^{(2)} = q_{41}^{'(2)}(1 - 0.5q_{41}^{'(1)}) = 0.11(1 - .5 * 0.00057) = 0.1099687.$$

It should then be straightforward to verify the entries in the following multiple decrement table. The age 40+ denotes immediately following the resignations.

x	$\ell_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
39	100,000	46.795	8,997.795	-
40	90,955.41			18,191.08
40+	72,764.33	36.63684	7,274.505	-
41	65,453.19	35.25636	7,197.799	-

(b) $$\frac{\ell_{42}^{(\tau)}}{\ell_{39}^{(\tau)}} = \frac{65453.19 - (35.25636 + 7197.799)}{100000} = \frac{58220.13}{100000} = 0.5822013$$

(c) $$\frac{d_{41}^{(2)}}{\ell_{39}^{(\tau)}} = \frac{7197.799}{100000} = 0.07197799$$

(d) Suppose D is the annual deposit required. Then the APV of these deposits will be equal to

$$D \times \left[1 + v\frac{\ell_{40}^{(\tau)}}{\ell_{39}^{(\tau)}} + v^2 \frac{\ell_{41}^{(\tau)}}{\ell_{39}^{(\tau)}}\right] = D \times \left[1 + (1/1.08)\frac{72764.33}{100000} + (1/1.08)^2 \frac{65453.19}{100000}\right] = D \times 2.234899$$

The APV of the transfer grant is equal to (assuming transfers occur at the end of each birthday/year)

$$\begin{aligned}
\text{APV(grant)} &= 10000 \times \left[v \frac{d_{39}^{(2)}}{\ell_{39}^{(\tau)}} + v^2 \frac{d_{40}^{(2)}}{\ell_{39}^{(\tau)}} + v^3 \frac{d_{41}^{(2)}}{\ell_{39}^{(\tau)}} \right] \\
&= 10000 \times \left[(1/1.08) \frac{8997.795}{100000} + (1/1.08)^2 \frac{7274.505}{100000} + (1/1.08)^3 \frac{7197.799}{100000} \right] \\
&= 2028.185
\end{aligned}$$

Solving for D, we get

$$D = \frac{2028.185}{2.234899} = 907.5063.$$