Exercise 8.17

(a) Assuming UDD in the multiple decrement model (the typical assumption made when converting multiple decrement rates to independent rates), this leads us to the familiar formula

$$q_x^{'(j)} = 1 - (1 - q_x^{(\tau)})^{q_x^{(j)}/q_x^{(\tau)}}.$$

Since

$$q_{40}^{(\tau)} = (2400 + 51)/15490 = 0.1582311,$$

it follows that

$$q_{40}^{'(2)} = 1 - (1 - 0.1582311)^{51/2451} = 0.003577730.$$

(b) If withdrawal occurs at exact age 40, then

$$q_{40}^{'(2)} = \frac{51}{15490 - 2400} = 0.003896104.$$