Exercise 8.4

(a) The reserve at time t for a policy in state 1 can be expressed as

$$\frac{d}{dt}\,{}_tV^{(1)} = \delta\,{}_tV^{(1)} - 50000 - \mu^{12}_{50+t}\left(200000 - {}_tV^{(1)}\right) - \mu^{10}_{50+t}\left({}_tV^{(0)} - {}_tV^{(1)}\right)$$

(b) The APV of future premiums is

$$\text{APV(FP)} = P\,\bar{a}^{00}_{50} = 11.9520\,P$$

and the APV of future sickness benefits is

$$\text{APV(FSB)} = 50000\,\bar{a}^{01}_{50} = 50000 \times 1.3292$$

and the APV of future death benefit is

$$\text{APV(FDB)} = 200000\,\bar{A}^{02}_{50} = 200000 \times 0.34980.$$

Solving for P, we get

$$P = 50000 \times \frac{1.3292 + 4 \cdot 0.34980}{11.9520} = 11,413.99.$$

(c) The reserve at time 10 for the policy in state 1 is then

$$\begin{aligned}
{}_{10}V^{(1)} &= \text{APV(FSB)}_{10} + \text{APV(FDB)}_{10} - \text{APV(FP)}_{10} \\
&= 50000\,\bar{a}^{11}_{60} + 200000\,\bar{A}^{12}_{60} - P\,\bar{a}^{10}_{60} \\
&= 50000(7.1596) + 200000(0.56316) - 11413.99(1.7922) \\
&= 450,155.80.
\end{aligned}$$