Exercise 9.15

The APV of future benefits at issue is equal to

$$\text{APV}(\text{FB}_0) = 500000 \times \int_0^\infty v^t \, {}_tp^{00}_{28:27} \big(\mu^{02}_{28+t:27+t} + \mu^{03}_{28+t:27+t}\big) dt,$$

where we know that

$${}_tp^{00}_{28:27} = \exp\left[-\int_0^t \big(\mu^{01}_{28+s:27+s} + \mu^{02}_{28+s:27+s} + \mu^{03}_{28+s:27+s}\big) ds \right].$$

Let P denote the annual net premium so that the APV of future premiums is equal to

$$\text{APV}(\text{FP}_0) = P \times \sum_{k=0}^{29} v^k \, {}_kp^{00}_{28:27}.$$

Solving for P, we get

$$P = 500000 \times \frac{\int_0^\infty v^t \, {}_tp^{00}_{28:27} \big(\mu^{02}_{28+t:27+t} + \mu^{03}_{28+t:27+t}\big) dt}{\sum_{k=0}^{29} v^k \, {}_kp^{00}_{28:27}} = 500000 \times \frac{0.1436813}{14.51844} = 4948.236.$$

The details of the calculations are written in R as shown below.

```
A <- 0.0001
B <- 0.0003
c <- 1.075
D <- 0.00035
mu01xy <- function(y){
A + B*c^y}
mu02xy <- function(x){
A + D*c^x}
mu03xy <- 5*(10^(-5))
tp00xy <- function(x,y,t){
temp1 <- A*t + B*c^y*(c^t-1)/log(c)
temp2 <- A*t + D*c^x*(c^t-1)/log(c)
exp(-temp1-temp2-mu03xy*t)}
i <- 0.05
v <- 1/(1+i)
delta <- log(1+i)
integ1 <- function(t){
v^t * tp00xy(28,27,t) * (mu02xy(28+t) + mu03xy)
}
APVben <- integrate(integ1,0,200)
k <- 0:29
APVprem <- sum(v^k * tp00xy(28,27,k))

> APVben$value
```

```
[1] 0.1436813

> APVprem
[1] 14.51844

> P
[1] 4948.236
```