Exercise 9.3

For Smith we are given that

$$\mu^S_{30+t} = Bc^{30+t} = 0.0003 \times 1.07^{30+t}$$

so that

$${}_tp^S_{30} = \exp\left\{-\left[\frac{Bc^{30}}{\log(c)}(c^t - 1)\right]\right\}.$$

For Jones, we are given

$$\mu^J_{30+t} = Bc^{30+t} + 0.039221 = \mu^S_{30+t} + 0.039221.$$

so that

$${}_tp^J_{30} = \mathrm{e}^{-0.039221t} \times {}_tp^S_{30}.$$

The probability that Jones dies before reaching age 50 and before Smith dies is given by

$${}_{20}q^{\;1}_{30:30} = \int_0^{20} {}_tp^J_{30}\, \mu^J_{30+t} \times {}_tp^S_{30}\, dt = 0.5673757.$$

For those interested in the simple R code to evaluate the integral above, we have

```
A <- 0.039221
B <- 0.0003
c <- 1.07
muS30 <- function(t){
out <- B*c^(30+t)
out}
muJ30 <- function(t){
out <- A + B*c^(30+t)
out}
tpS30 <- function(t){
temp <- B*c^30*(c^t - 1)/log(c)
out <- exp(-temp)
out}
tpJ30 <- function(t){
temp <- A*t + B*c^30*(c^t - 1)/log(c)
out <- exp(-temp)
out}
h <- 1/1000
t <- seq(0,20,h)
intJS <- tpJ30(t)*muJ30(t)*tpS30(t)
probJS <- 0
n <- 1
while (n<length(t)) {
n <- n+2
```

```
probJS <- probJS + (h/3)*(intJS[n-2]+4*intJS[n-1]+intJS[n])
}
probJS

> probJS
[1] 0.5673757
```