Exercise 9.9

Here we consider the joint life (25:25) where the first age refers to that of Bob, and the second, that of Mike.

(a) Let P denote the annual net premium for the policy. The APV of future premiums is therefore

$$\text{APV}(\text{FP}_0) = P\,\ddot{a}_{25},$$

where here life (25) refers to that of Bob. The APV of future benefits can be expressed as

$$\text{APV}(\text{FB}_0) = 100000\,A^{\;\;2}_{25:25}$$

and because mortality follows Gomertz's, we can use the result of Exercise 8.11:

$$\begin{aligned} A^{\;\;2}_{25:25} &= A_{25} - A^{\;\;1}_{25:25} \\ &= A_{25} - \frac{c^{25}}{c^w} A_w, \end{aligned}$$

where the joint life (25:25) is replaced by a single life (w) that satisfies

$$w = \frac{\log\left(2c^{25}\right)}{\log(c)} = 34.58436.$$

Solving for P, we therefore have

$$\begin{aligned} P &= 100000 \times \frac{A_{25} - \left(c^{25}/c^w\right) A_w}{\ddot{a}_{25}} \\ &= 100000 \times \frac{0.152806 - (6.09834/12.19668)(0.1095454)}{17.79107} \\ &= 243.159 \end{aligned}$$

```
B <- 0.0003
c <- 1.075
muxt <- function(x,t){
B*c^(x+t)}
tpx <- function(x,t){
temp <- B*c^x*(c^t-1)/log(c)
exp(-temp)}
i <- 0.05
v <- 1/(1+i)
d <- 1-v
# limiting age
ww <- 143
tt <- ww-25
t <- 0:tt
vt <- v^t
```

```
ann25 <- sum(vt*tpx(25,t))
A25 <- 1-d*ann25
w <- log(2*c^25)/log(c)
annw <- sum(vt*tpx(w,t))
Aw <- (c^25/c^w)*(1-d*annw)
P <- 100000*(A25 - Aw)/ann25

> A25
[1] 0.152806
> w
[1] 34.58436
> c^25
[1] 6.09834
> c^w
[1] 12.19668
> Aw
[1] 0.1095454
> ann25
[1] 17.79107
> P
[1] 243.159
```

(b) To calculate the policy value at the end of 10 years, we consider the two cases:

(i) Bob is alive while Mike is dead. The policy value then is

$$ {}_{10}V = 100000\, A_{35} - P\, \ddot{a}_{35} = 100000(0.2224011) - 243.159(16.32958) = 18269.42. $$

```
tt <- ww-35
t <- 0:tt
vt <- v^t
ann35 <- sum(vt*tpx(35,t))
A35 <- 1-d*ann35
V10a <- 100000*A35 - P*ann35

> A35
[1] 0.2224011
> ann35
[1] 16.32958
> V10a
[1] 18269.42
```

(ii) Both Bob and Mike are alive. The policy value then is

$$ \begin{aligned} {}_{10}V &= 100000\left[A_{35} - (c^{25}/c^{w})A_w\right] - P\, \ddot{a}_{35} \\ &= 100000[0.2224011 - (12.56887/25.13774)(0.3090295)] - 243.159(16.32958) \\ &= 2817.949. \end{aligned} $$

```
w <- log(2*c^35)/log(c)
annw <- sum(vt*tpx(w,t))
Aw <- 1-d*annw
V10b <- 100000*(A35 - (c^35/c^w)*Aw) - P*ann35

> w
[1] 44.58436
> c^35
[1] 12.56887
> c^w
[1] 25.13774
> Aw
[1] 0.3090295
> V10b
[1] 2817.949
```