4.4 Problems

Question 1. Evaluate the general indefinite integral.

(a) $\int (u+2)(3-u)\ du$

$= \int (-u^2+u+6)\,du$

$= -\frac{1}{3}u^3 + \frac{1}{2}u^2 + 6u + C$

(b) $\int \frac{\sin 2x}{\cos x} dx \quad = \int \frac{2\sin x \cancel{\cos x}}{\cancel{\cos x}}\,dx$

$= -2\cos x + C$

Question 2. Verify that the formula: $\int \cos^3 x\ dx = \sin x - \frac{1}{3}\sin^3 x + C$ is correct.

$\frac{d}{dx}\left(\sin x - \frac{1}{3}\sin^3 x + C\right) = \cos x - \sin^2 x \cdot \cos x$

$= \cos x (1 - \sin^2 x)$

$= \cos x \cdot \cos^2 x$

$= \cos^3 x$

so the antiderivative of $\cos^3 x$ is $\sin x - \frac{1}{3}\sin^3 x + C$

Question 3. If $h'(t)$ the the rate of growth of Ryan in inches/year what does $\int_5^8 h'(t)\,dt$ represent?

the total change of Ryan's height from years 5 to year 8

Question 4. Water flows from the bottom of a storage tank at a rate of $r(t) = 10 - 2t$ liters per minute, where $0 \leq t \leq 5$.

(a) After a minute a 10 liter bucket is placed under the storage tank to catch the water. How long until the bucket starts to overflow?

$$10 = \int_1^b (10-2t)\,dt = (10t - t^2)\Big|_1^b = 10b - b^2 - (10-1)$$

$$\Leftrightarrow \quad b^2 - 10b + 19 = 0$$

$$b = \cancel{5+\sqrt{6}} \text{ or } 5-\sqrt{6} \text{ min}$$

(b) At $t = 4$ another 10 liter bucket is placed under the storage tank to catch the water. How much water does this bucket have in it at the end? $t=5$

$$\text{volume of water at the end} = \int_4^5 (10-2t)\,dt$$

$$= (10t - t^2)\Big|_4^5$$

$$= 50 - 25 - (40-16)$$

$$= 1 \text{ liter}$$