4.5a Problems

Question 1. Evaluate

(a) $\int (2+3x)^8 \, dx$

$u = 2+3x$

$du = 3dx$

$\frac{du}{3} = dx$

$\int u^8 \frac{du}{3}$

$= \frac{1}{27}u^9 + C$ change back to x

$= \frac{1}{27}(2+3x)^9 + C$

(b) $\int \sin x \cos x \, dx$

$= \int \frac{1}{2} \sin 2x \, dx$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

$= \int \frac{1}{2} \sin u \frac{du}{2}$

$= -\frac{\cos u}{4} + C$

$= -\frac{\cos 2x}{4} + C$

(c) $\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}}$

$u = 1+\tan t$

$du = \sec^2 t \, dt$

$\frac{du}{\sec^2 t} = dt$

$= \int \frac{du}{\cos^2 t \sec^2 t \sqrt{u}}$

$= 2u^{\frac{1}{2}} + C$

$= 2\sqrt{1+\tan t} + C$

Question 2. Evaluate

(a) $\int x^2\sqrt{x+2}\,dx$

$x+2=u \Rightarrow x=u-2$

$dx=du$

$= \int (u-2)^2\sqrt{u}\,du$

$= \int (u^2-4u+4)\sqrt{u}\,du$

$= \int \left(u^{5/2}-4u^{3/2}+4u^{\frac{1}{2}}\right)du$

$= \frac{2}{7}u^{7/2} - 4\cdot\frac{2}{5}u^{5/2} + 4\cdot\frac{2}{3}u^{3/2} + C = \frac{2}{7}(x+2)^{7/2} - \frac{8}{5}(x+2)^{5/2} + \frac{8}{3}(x+2)^{3/2} + C$

(b) $\int x^3\sqrt{x^2+1}\,dx$

$x^2+1=u \Rightarrow x^2=u-1$

$2x\,dx=du$

$dx=\frac{du}{2x}$

$= \int x^3\sqrt{u}\,\frac{du}{2x}$

$= \int \frac{x^2}{2}\sqrt{u}\,du$

$= \int \frac{u-1}{2}\sqrt{u}\,du$

$= \frac{1}{2}\int \left(u^{3/2}-u^{1/2}\right)du$

$= \frac{1}{2}\left(\frac{2}{5}\cdot u^{5/2} - \frac{2}{3}u^{3/2}\right) + C = \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$

(c) $\int \sin t(1-\sin^2 t)^2\,dt$ (Hint: Trigonometric properties FTW)

$= \int \sin t \cos^4 t\,dt$

$u=\cos t$

$du=-\sin t\,dt$

$\frac{du}{-\sin t}=dt$

$= \int -u^4\,du$

$= -\frac{1}{5}u^5 + C$

$= -\frac{1}{5}\cos^5 t + C$

Question 3. Find the net area under the curve $2 + \cos(\pi t/2)$ between $x = 0$ and $x = 3$

$$\int_0^3 (2 + \cos \pi t/2)\, dA \qquad u = \frac{\pi t}{2} \qquad \text{when } t=0,\ u=0$$

$$du = \frac{\pi}{2}\, dt \qquad \text{when } t=3,\ u = \frac{3\pi}{2}$$

$$= \int_0^{\frac{3\pi}{2}} (2 + \cos u)\, \frac{2}{\pi}\, du \qquad \frac{2}{\pi}\, du = dt$$

$$= \left(\frac{4}{\pi} u + \frac{2}{\pi} \sin u\right)\Big|_0^{\frac{3\pi}{2}}$$

$$= \frac{4}{\pi} \cdot \frac{3\pi}{2} + \frac{2}{\pi} \sin \frac{3\pi}{2} - 0 = 6 - \frac{2}{\pi}$$

Question 4. Calculate

(a) $\displaystyle\int_{1/2}^{1} \frac{\sin(x^{-2})}{x^3}\, dx$

$$u = x^{-2} \qquad \text{when } x = \frac{1}{2},\ u = 4$$

$$du = -2x^{-3}\, dx \qquad \text{when } x = 1,\ u = 1$$

$$= \int_4^1 \frac{\sin(u)}{x^3}\, x^3\, \frac{du}{-2} \qquad \frac{du}{-2}\, x^3 = dx$$

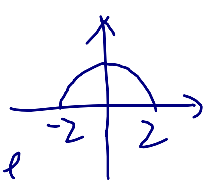
$$= \int_4^1 \frac{\sin u}{-2}\, du$$

$$= \frac{\cos u}{2}\Big|_4^1 = \frac{\cos 1 - \cos 4}{2}$$

(b) $\displaystyle\int_{-2}^{2} (x+3)\sqrt{4 - x^2}\, dx$ (Hint: Algebra then Geometry FTW)

$$= \underbrace{\int_{-2}^{2} x\sqrt{4-x^2}\, dx}_{\text{odd function}} + \int_{-2}^{2} 3\sqrt{4-x^2}\, dx$$

↑ half a circle centered at (0,0) with radius 2 so area is 2π



$$= 0 + 6\pi$$

$$= 6\pi$$

Question 5. If f is continuous $\int_0^4 f(x)\,dx = 10$, then $\int_0^2 f(2x)\,dx$ $= \int_0^4 f(u)\,\frac{du}{2} = 5$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

A. 40

B. 20

C. 10

D. 5

E. None of the above

Question 6. If f is continuous $\int_0^9 f(x)\,dx = 4$, then $\int_0^3 x f(x^2)\,dx$ $= \int_0^9 \frac{1}{2} f(u)\,du = 2$

$u = x^2$

$du = 2x\,dx$

A. 8

B. 4

C. 2

D. 1

E. None of the above