Name: ______________________ **PID:** ______________

1. (5 points) a) Find $\frac{dy}{dx}$ from $y^2 + 13x = x^2y + 13$ using implicit differentiation.
   b) Find the equation of the tangent line at the point $(4, 3)$.

$$2yy' + 13 = 2xy + x^2y'$$

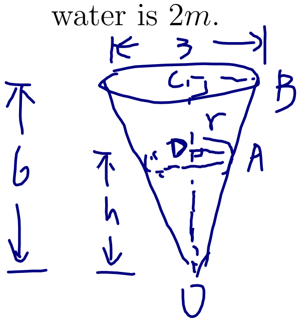
$$\Rightarrow (2y - x^2)y' = 2xy - 13$$

$$\Rightarrow y' = \frac{2xy - 13}{2y - x^2}$$

at $(4,3)$ $y' = \frac{2 \cdot 3 \cdot 4 - 13}{2 \cdot 3 - 4^2} = -\frac{11}{10}$

The tangent line is $y - 3 = -\frac{11}{10}(x - 4)$

2. (4 points) Water is leaking out of an inverted conical tank at a rate of $10m^3/min$. The tank has height $6m$ and the diameter at the top is $3m$. Find how fast the water level is dropping when the height of the water is $2m$.



$V' = -10$ $h'(t_0) = ?$ $h(t_0) = 2$

$$V = \frac{1}{3}\pi r^2 h \quad (1)$$

Method 1, simplify (1) first:

ΔOAD and ΔOCB are similar triangles.

So $\frac{AD}{CB} = \frac{OD}{OC} \Leftrightarrow \frac{r}{3/2} = \frac{h}{6} \Rightarrow r = \frac{h}{4}$

Plug $r = \frac{h}{4}$ into (1) we get $V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h$

$$= \frac{1}{48}\pi h^3 \quad (2)$$

Differentiate (2). $V' = \frac{1}{16}\pi h^2 h'$

at $t = t_0$: $-10 = \frac{1}{16}\pi 2^2 h' \Rightarrow h' = \frac{-40}{\pi} m/min$

Method 2: differentiate (1)

$$V' = \frac{1}{3}\pi(2r \cdot r'h + r^2h') \quad (3)$$

At $t = t_0$, $-10 = \frac{1}{3}\pi(2r \cdot r' \cdot 2 + r h')$

need to find $r(t_0)$ and $r'(t_0)$ to get $h'(t_0)$

Since OAD and OCB are similar triangles

$\frac{r}{3/2} = \frac{h}{6} \Rightarrow r = h/4 \Rightarrow r' = h'/4$

plug these into (3)

$$-10 = \frac{1}{3}\pi\left(2 \cdot \frac{2}{4} \cdot \frac{h'}{4} \cdot 2 + \left(\frac{2}{4}\right)^2 h'\right)$$

$$\Rightarrow h' = -\frac{40}{\pi} m/min$$