

Can AI discover the drugs of the future?

Guo-Wei Wei

Mathematics

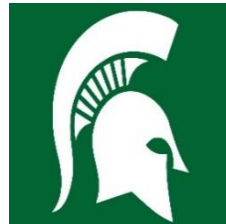
Michigan State University

<http://www.math.msu.edu/~wei>

The 8th International Congress of Chinese Mathematicians of 2019

Beijing, June 9 - 15, 2019

Grant support: NSF, NIH, MSU, BMS, and Pfizer



The Biggest Crisis of the Contemporary Science

The number of researchers in the world who know both graduate-level mathematics and molecular-level biology is smaller than the number of fields medalists!

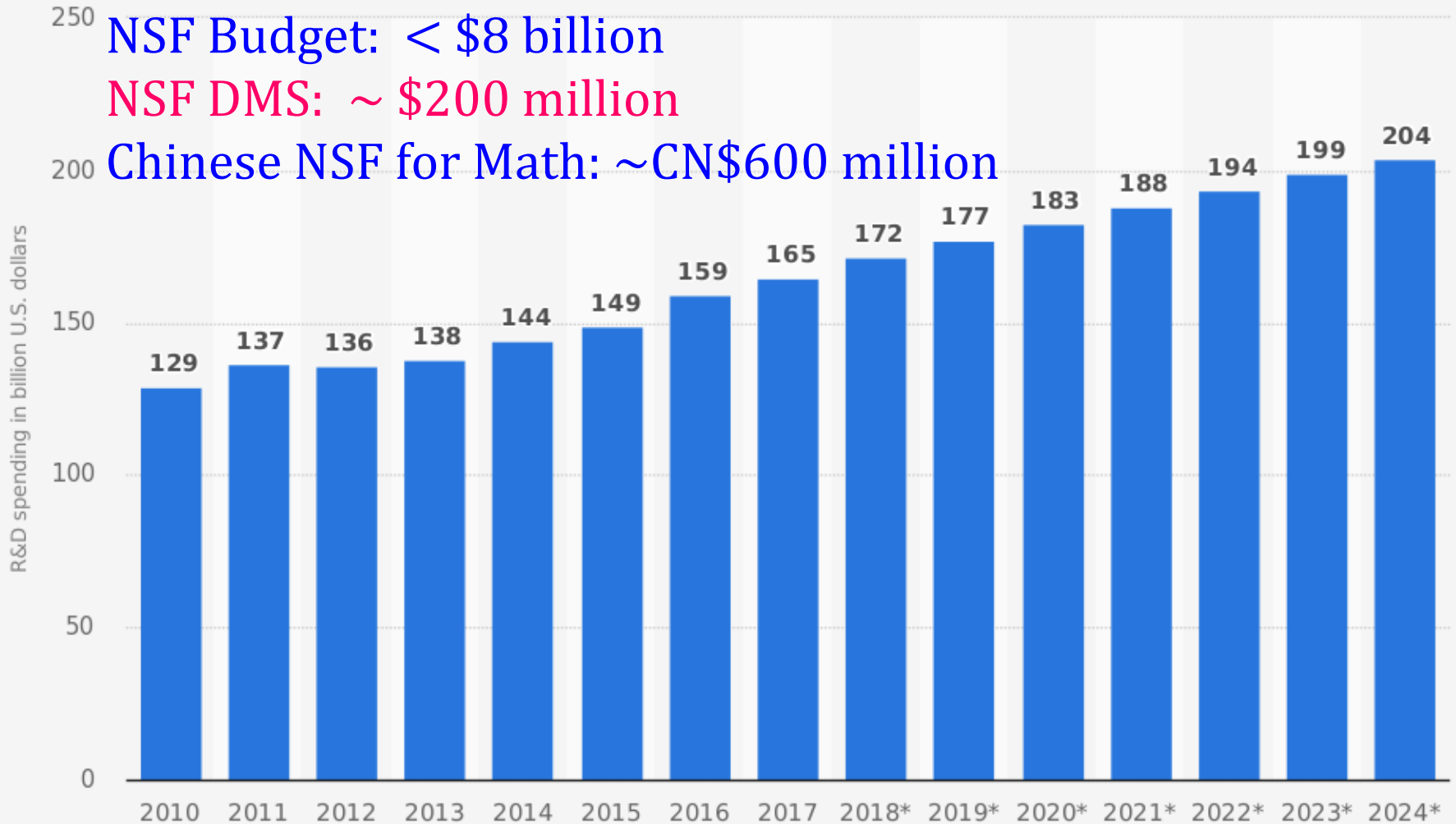
The rule of life has been there for billions of years but very little is known about it!

None knows the existence and uniqueness of mathematical foundation for life!

A Brief Summary of Modern Biological Science

1960	2000	2019
Organismal biology (i.e., nonliving organisms, living organisms, developmental biology, morphology, anatomy, physiology, and medicine)	Molecular organismal biology, organomics, connectomics, foodomics, physiomics, pharmacogenomics, ...	
Ecology	Molecular ecology	
Evolution (i.e., life, and evolutionary biology)	Molecular evolution	
Molecular and cellular biology (i.e., cell biology, biochemistry, molecular biology, and genetics)	Omics (e.g., genomics, proteomics, metabolomics, metagenomics, lipidomics, glycomics, transcriptomics, epigenomics, ...)	
Macroscopic	Mesoscopic	Microscopic

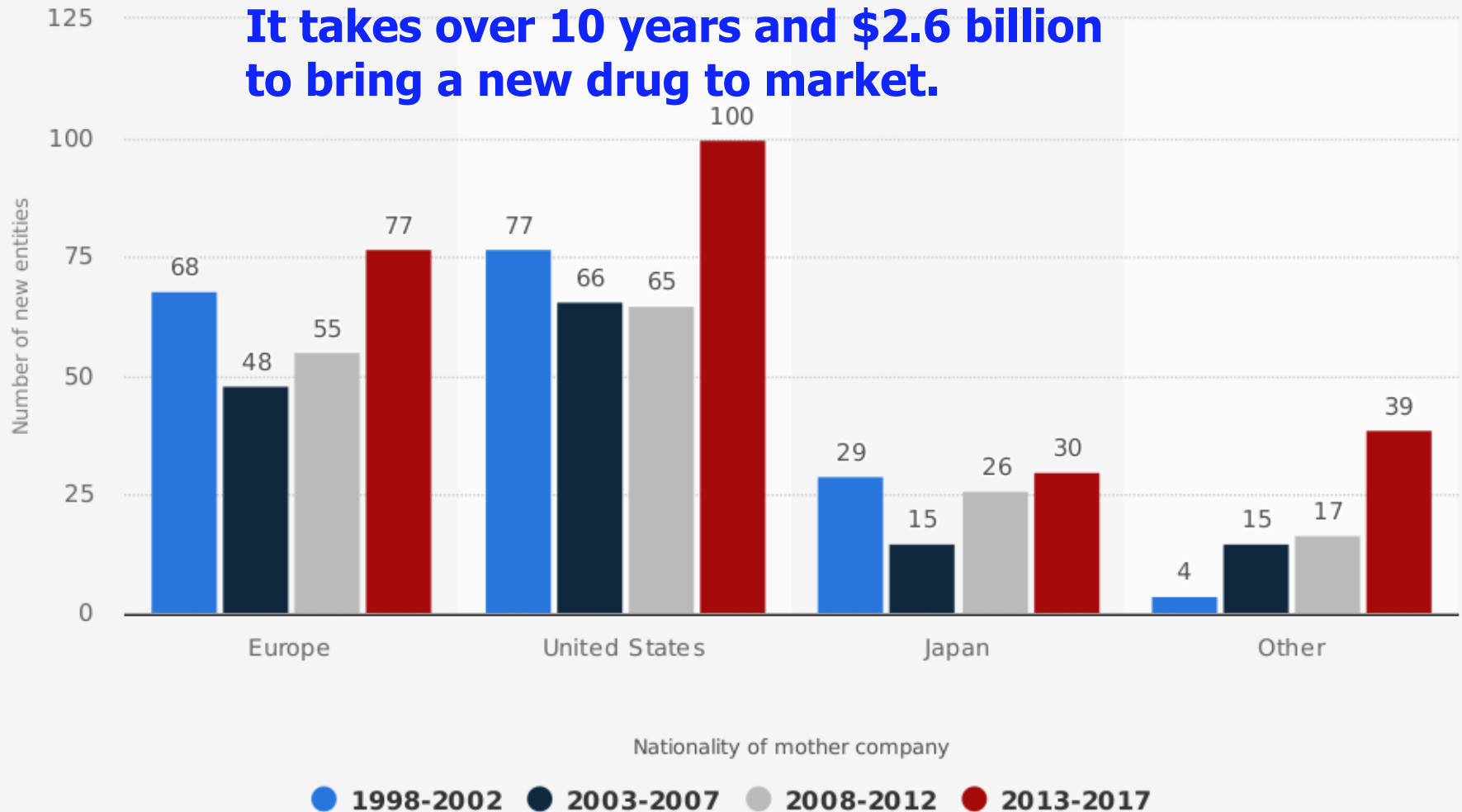
Total global spending on pharmaceutical research and development from 2010 to 2024 (in billion U.S. dollars)



Funding for mathematical research is $\sim \varepsilon^3$

Number of new chemical or biological entities developed between 1992 and 2017, by region of origin

It takes over 10 years and \$2.6 billion to bring a new drug to market.



Drug design and discovery

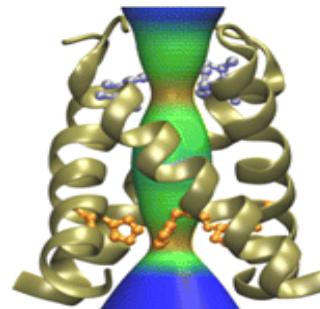


- 1) **Disease identification (physiology)**
- 2) **Target hypothesis (biochem./mole. biol.)**
- 3) **Virtual screening: drug pose, binding affinity, solubility, partition coefficient, toxicity, and side-effects (biophysics/bioinformatics)**
- 4) **Drug structural optimization in the target binding site (biochemistry/biophysics/synthetic chem.)**
- 5) **Preclinical *in vitro* and *in vivo* test**
- 6) **Clinical trials**
- 7) **Optimize drug's efficacy, pharmacokinetics, and pharmacodynamics properties (quantitative systems pharmacology)**

Influenza -- flu virus



M2 channel



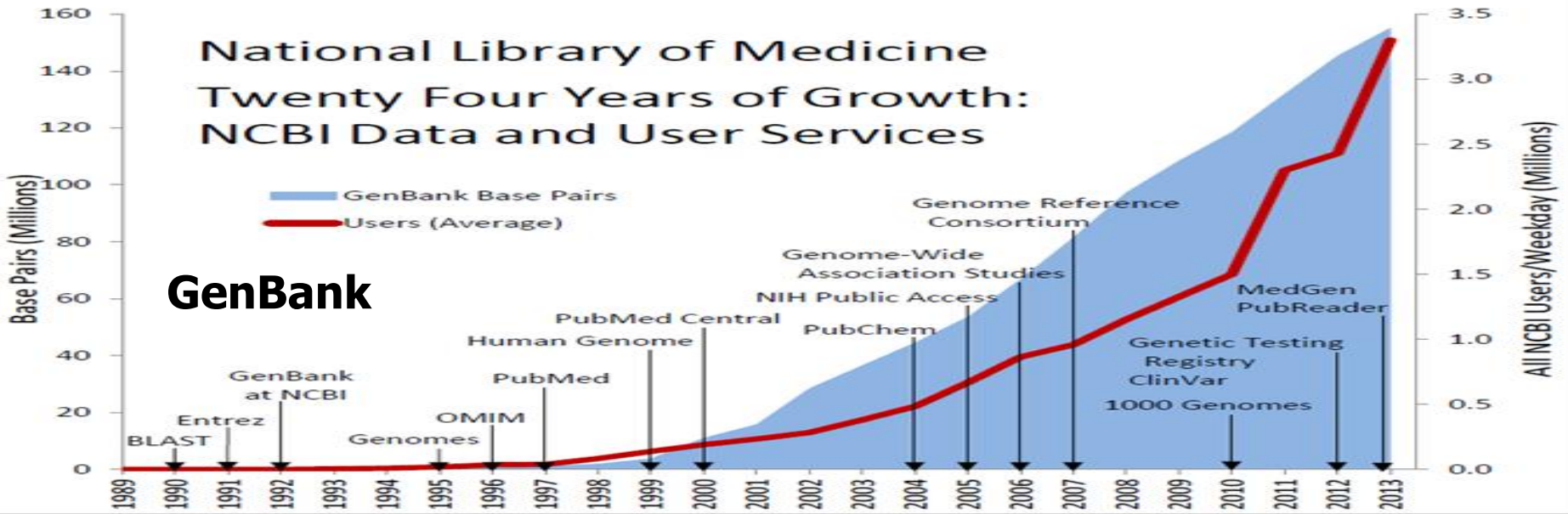
Amantadine M2-A complex



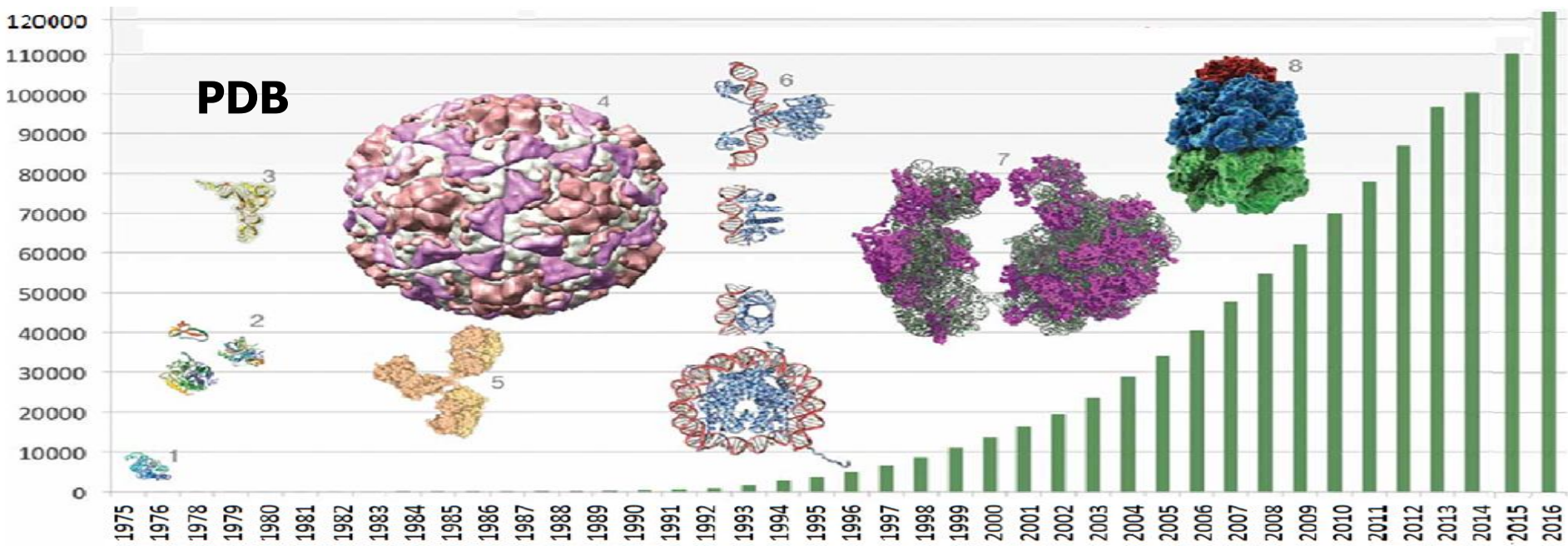
Biological data

National Library of Medicine Twenty Four Years of Growth: NCBI Data and User Services

GenBank

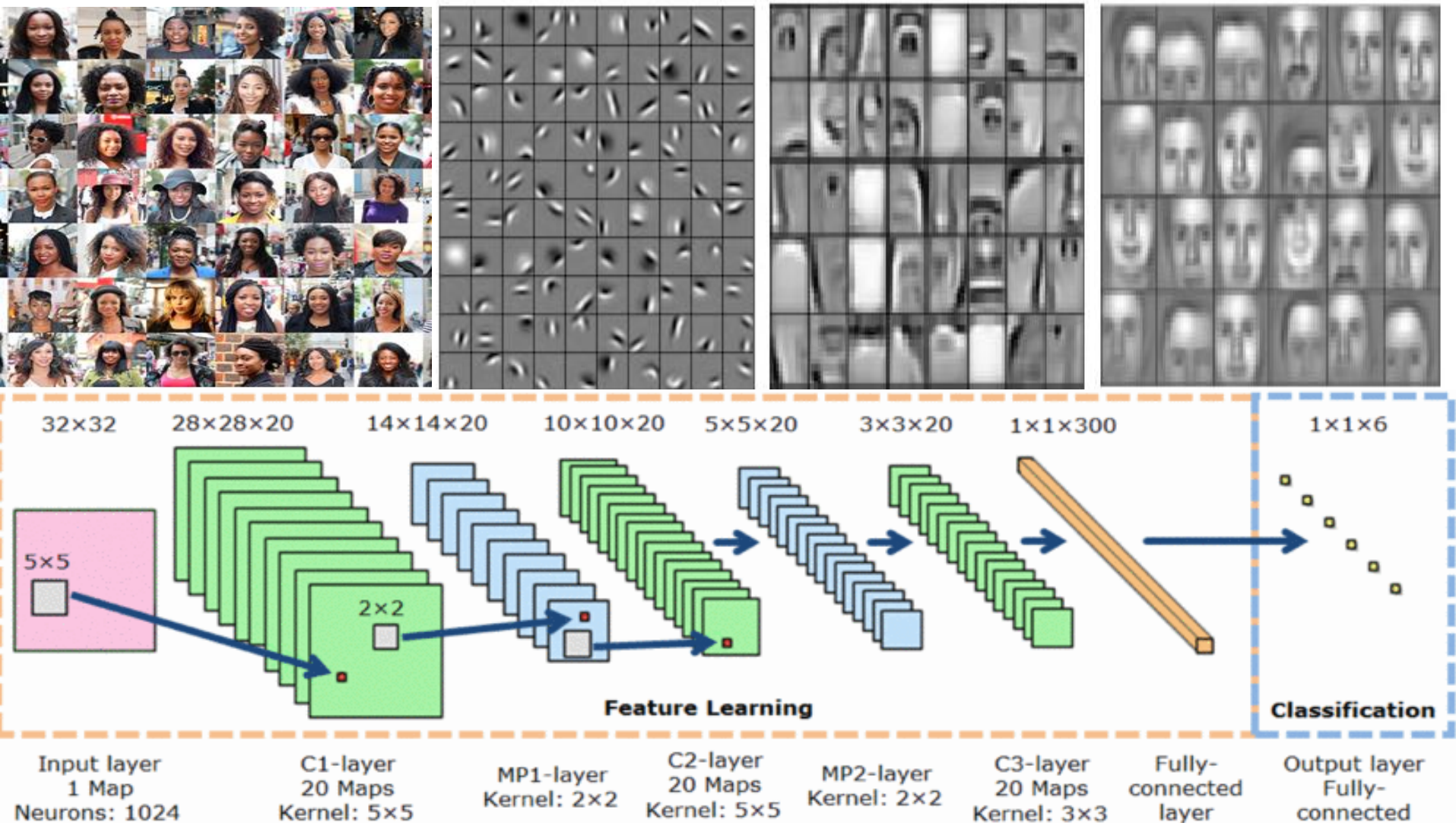


PDB



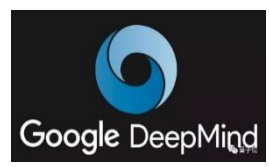
Artificial Intelligence & Deep learning

Bryson and Ho (Backpropagation 1969); Fukushima (Neo-Cognitron 1980); LeCun (CNN 1998); Hopfield (RNN 1982); Hochreiter and Schmidhuber (LSTM 1997); Goodfellow et al (GAN 2014); Autoencoder; Image translation, ...



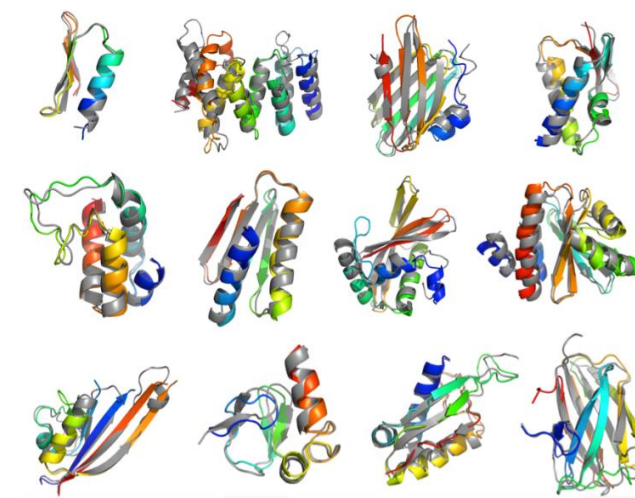
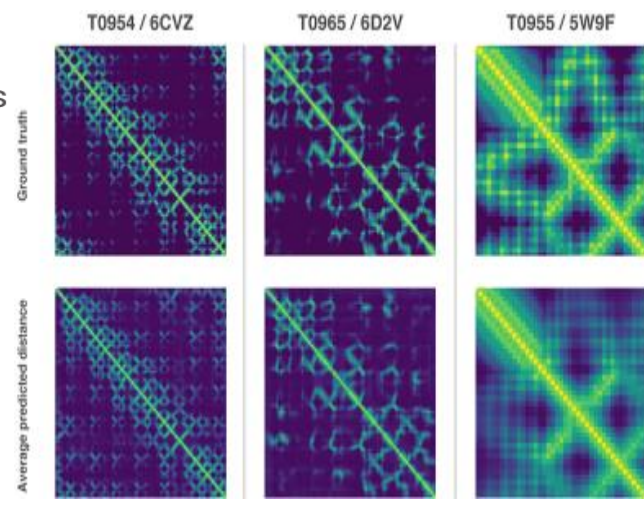
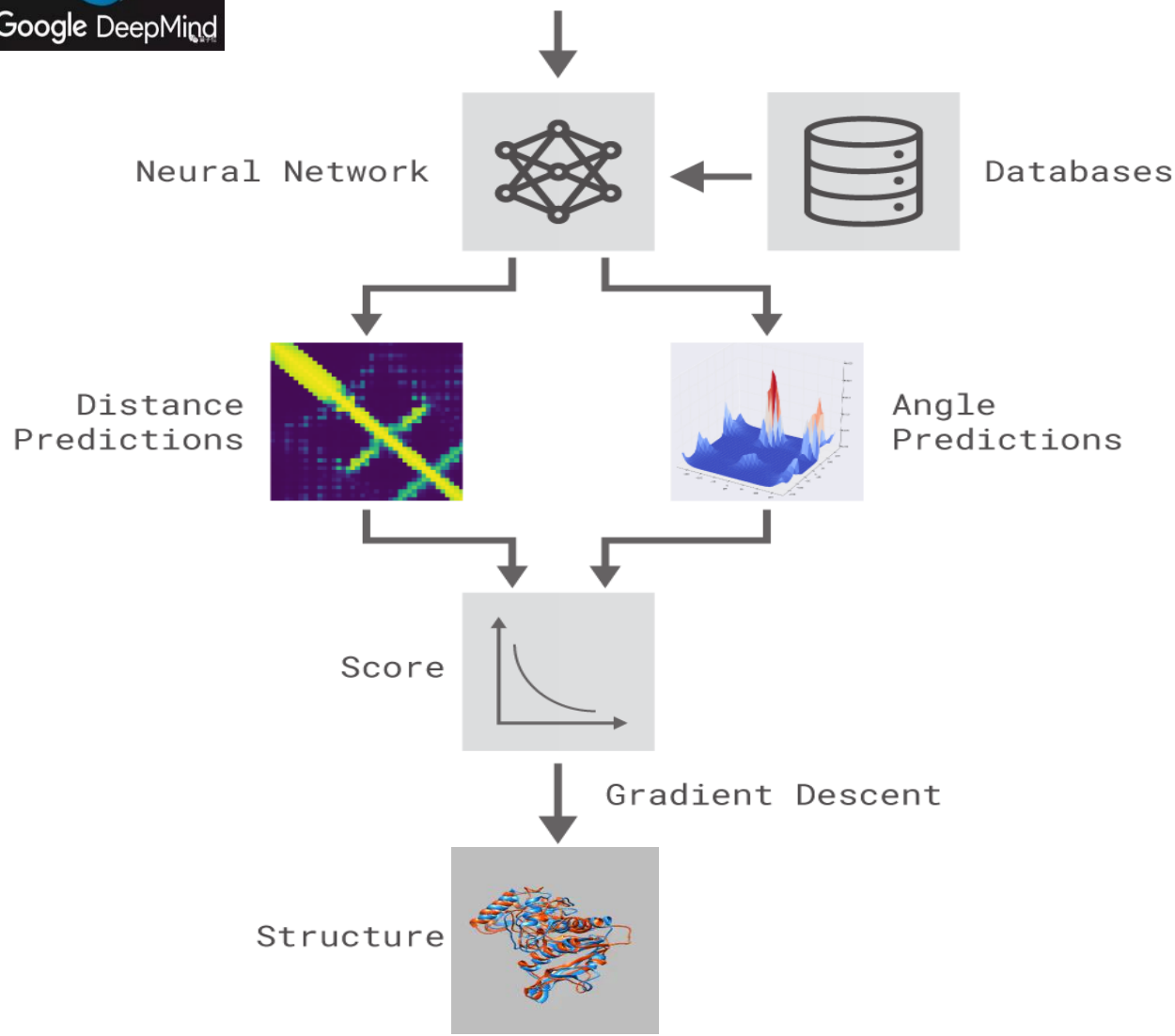
ALPHA FOLD

won 25 of 43 contests and was ranked 1st among 98 competitors in CASP 13.



Protein Sequence

SQETRKKCTEMKKKFKNCEVRCDESNHCVVRCSDTKYTLG



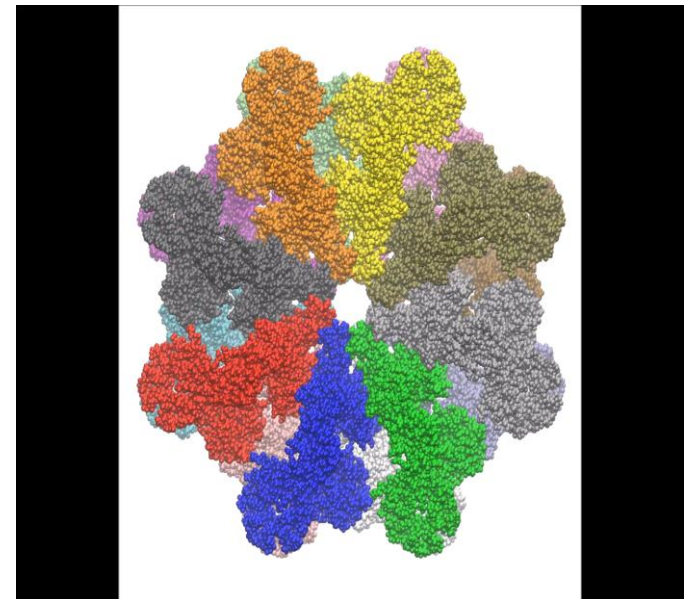
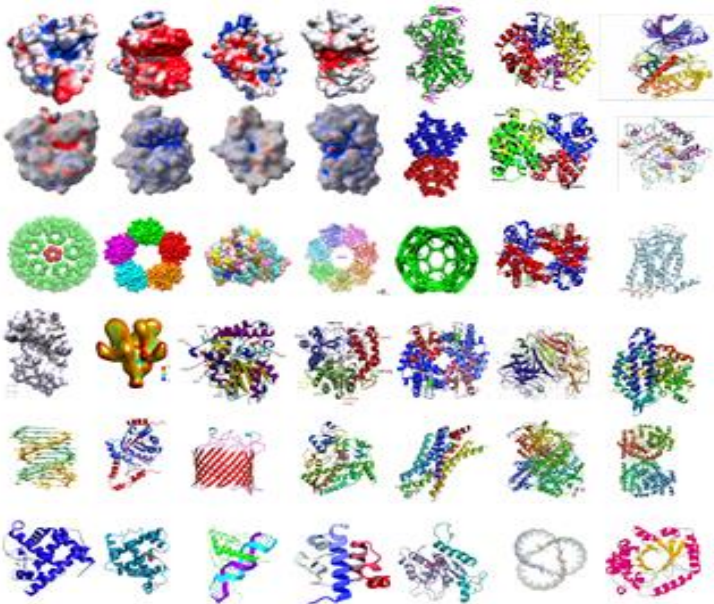
How to do deep learning for 3D biomolecular data?

Obstacles for deep learning of 3D biomolecules:

- **Geometric dimensionality:** \mathbb{R}^{3N} , where $N \sim 5000$ for a protein.
- **Machine learning dimensionality:** $> 1024^3 m$, where m is the number of atom types in a protein.
- **Molecules have different sizes --- non-scalable.**
- **Complexity:** intermolecular & intramolecular interactions

Solution:

- **Geometric simplification, dimension reduction & scale unification**





Sequence data
Structure data
Biophysics
Bioinformatics
Systems biology
Systems physiology

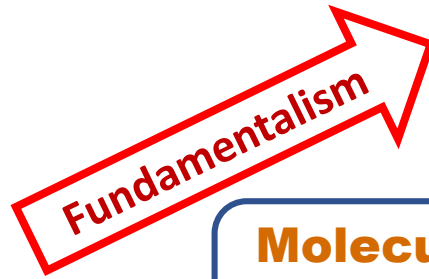
**Drug
Design &
Discovery**

Algebraic topology
Differential geometry
Graph theory
Multiscale modeling

Machine learning
Deep learning
Manifold learning
Transfer learning

Two schools of thinking

Given a protein with N atom and an average of n electrons in each atom

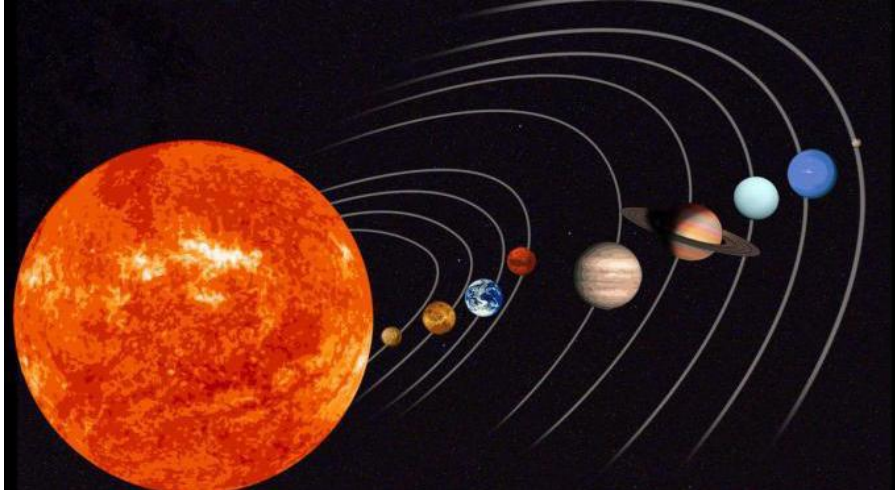


Quantum Mechanics
 \mathbb{R}^{3Nn+3N}

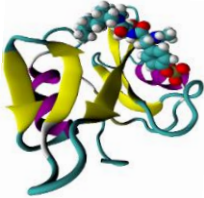
QM/MM \mathbb{R}^K
 $3N < K < 3N(n+1)$

Molecular Mechanics
 \mathbb{R}^{3N}

Multiscale Coarse-grain
 \mathbb{R}^M ($3 < M < 3N$)



Poisson-Boltzmann, PNP, etc. \mathbb{R}^3



Differentiable Manifold
 \mathbb{R}^2

Algebraic Topology
 \mathbb{R}^1

Graph Theory
 \mathbb{R}^0

Geo-Top Indices
 \mathbb{R}^0

Basic hypothesis:
Intrinsic physics lies on low-dimensional manifolds in a high dimensional space

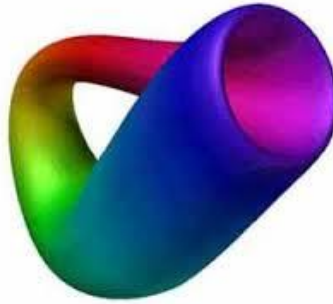


Classical Topology

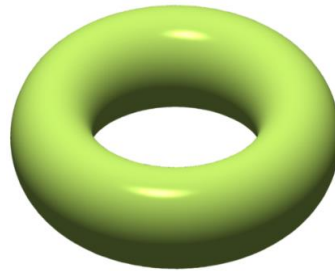
Möbius Strips (1858)



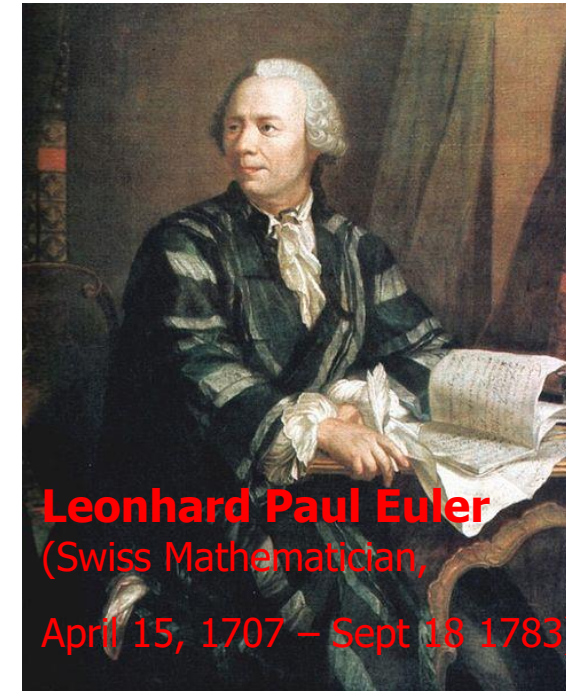
Klein Bottle (1882)



Torus



Double Torus



Leonhard Paul Euler
(Swiss Mathematician,
April 15, 1707 – Sept 18 1783)

**Seven Bridges
of Königsberg**



Leonhard Euler (1735)

"For the greatest benefit to mankind"
Augustus Voluc
The Royal Swedish Academy of Sciences has decided to award the
2016 NOBEL PRIZE IN PHYSICS



David J. Thouless
F. Duncan M. Haldane
J. Michael Kosterlitz

*"for theoretical discoveries of topological phase transitions
and topological phases of matter"*

Nobelprize.org

**Augustin-Louis Cauchy,
Ludwig Schläfli,
Johann Benedict Listing,
Bernhard Riemann, and
Enrico Betti**

Topological invariants: Betti numbers

β_0 is the number of connected components.

β_1 is the number of tunnels or circles.

β_2 is the number of cavities or voids.

Point

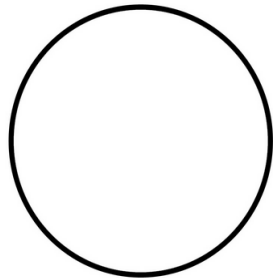


$$\beta_0 = 1$$

$$\beta_1 = 0$$

$$\beta_2 = 0$$

Circle

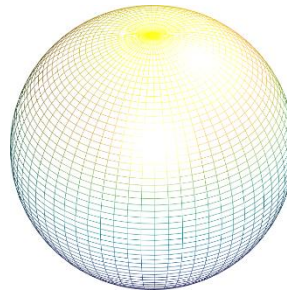


$$\beta_0 = 1$$

$$\beta_1 = 1$$

$$\beta_2 = 0$$

Sphere

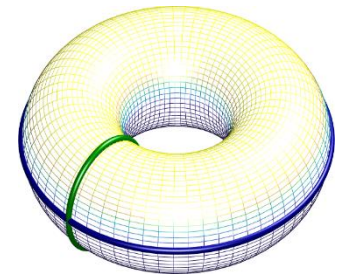


$$\beta_0 = 1$$

$$\beta_1 = 0$$

$$\beta_2 = 1$$

Torus



$$\beta_0 = 1$$

$$\beta_1 = 2$$

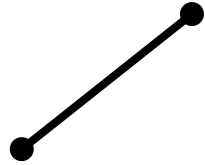
$$\beta_2 = 1$$

Vietoris-Rips complexes of planar point sets

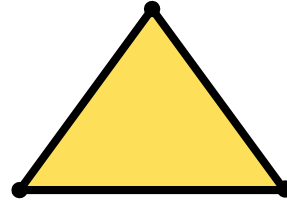
Simplexes:



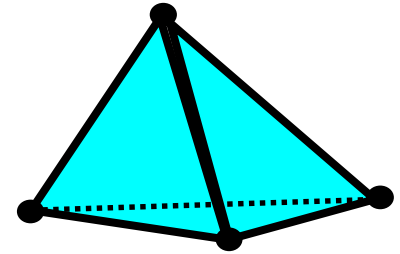
0-simplex



1-simplex

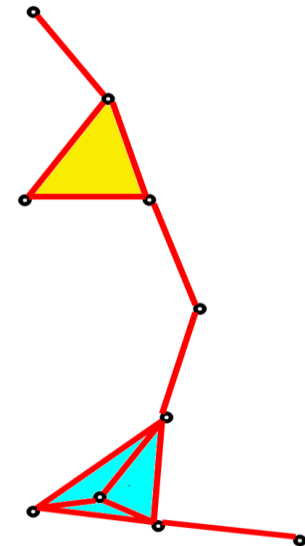
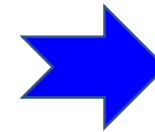
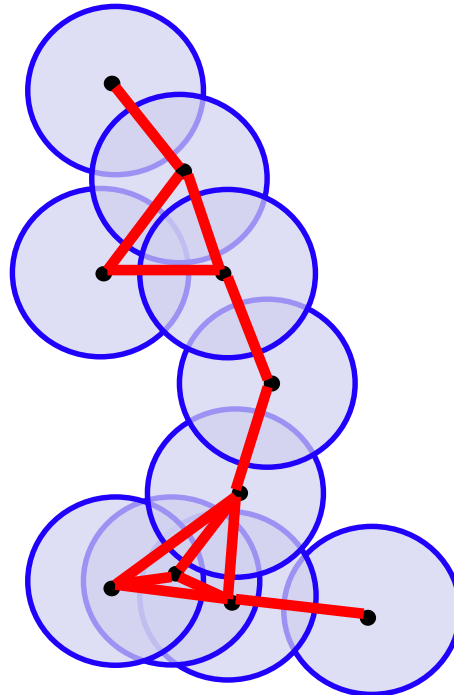
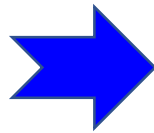
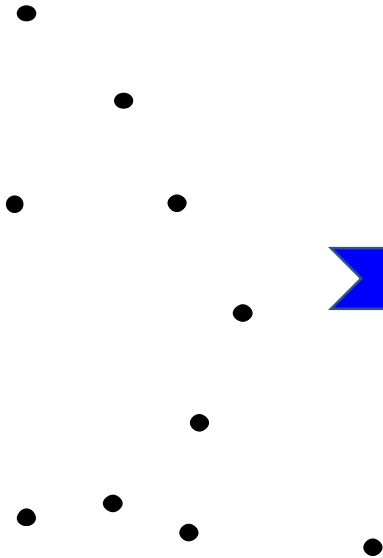


2-simplex



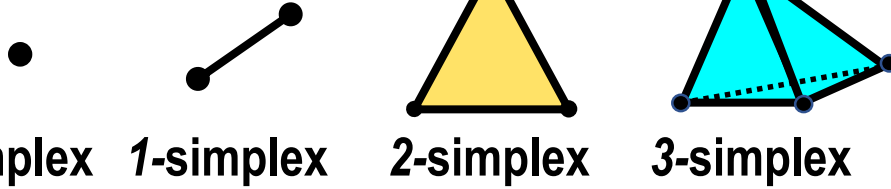
3-simplex

Simplicial complexes of ten points:



Persistent homology

Simplexes:



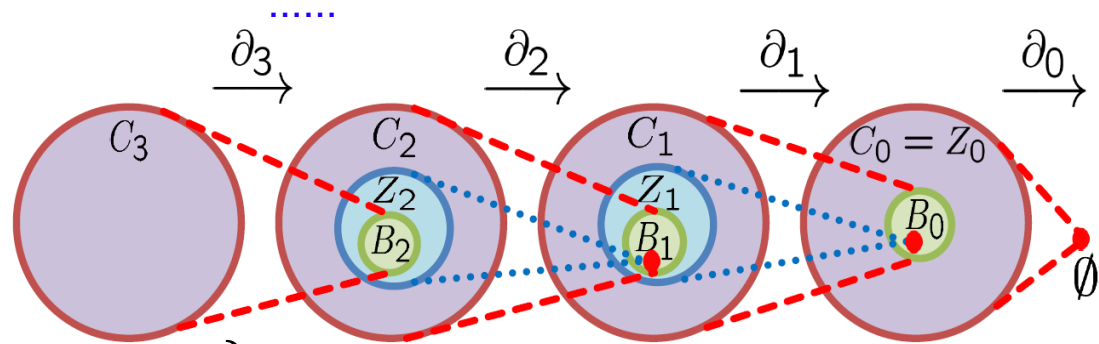
Frosini and Nandi (1999), Robins (1999), Edelsbrunner, Letscher and Zomorodian (2002), Zomorodian and Carlsson (2005), Edelsbrunner and Harer, (2007) Kaczynski, Mischaikow and Mrozek (2004), Ghrist (2008),

k-chain: $K = \left\{ \sum_j c_j \sigma_j^k \right\}$

Chain group: $C_k(K, \mathbb{Z}_2)$

Boundary operator:

$$\partial_k \sigma^k = \sum_{j=0}^k (-1)^j \{v_0, v_1, \dots, \hat{v}_j, \dots, v_k\}$$

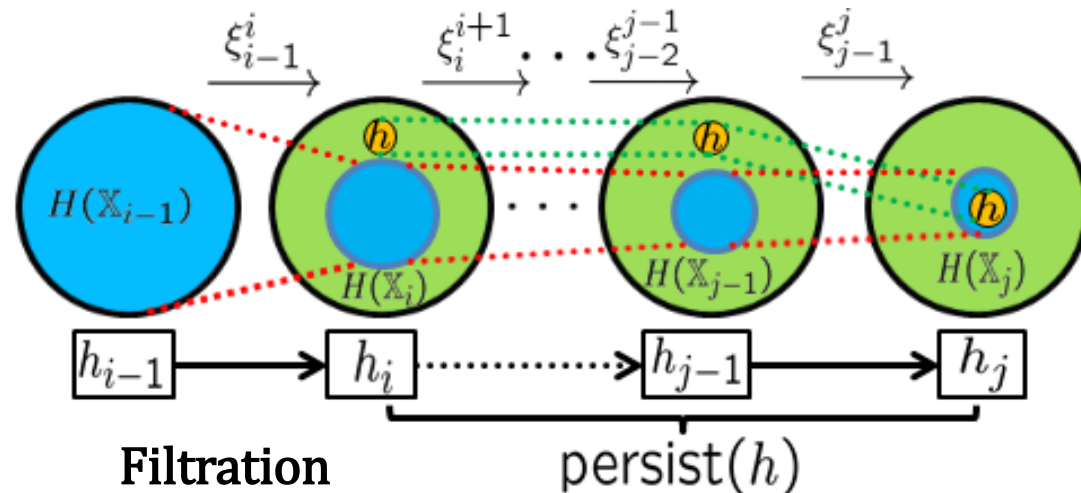


Cycle group: $Z_k = \text{Ker } \partial_k$

Boundary group: $B_k = \text{Im } \partial_{k+1}$

Homology group: $H_k = \frac{Z_k}{B_k}$

Betti number: $\beta_k = \text{Rank}(H_k)$



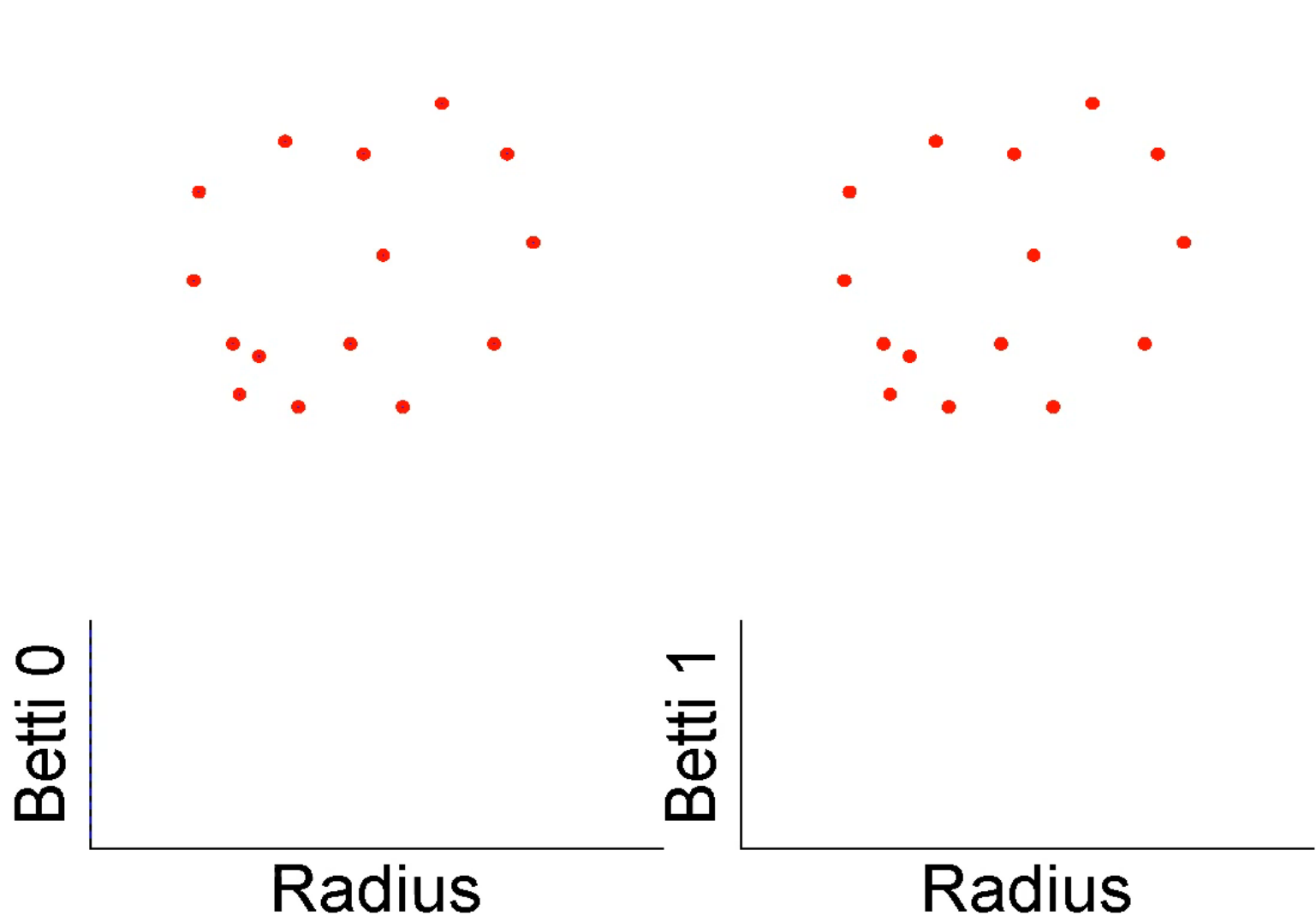
Xia, Wei, IJNMBE, 2014;

Xia, Feng, Tong, Wei, JCC, 2015

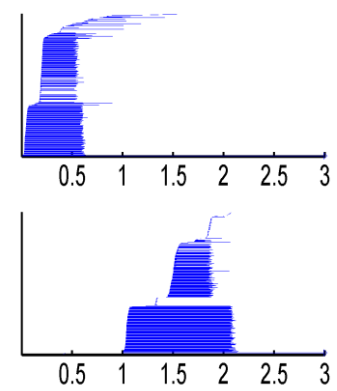
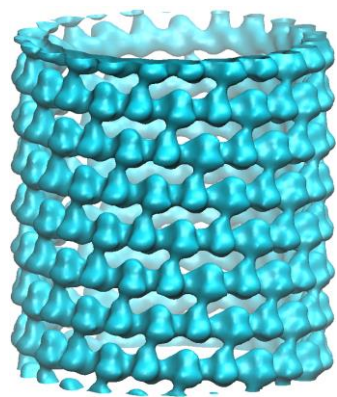
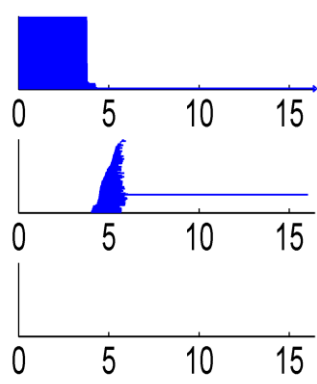
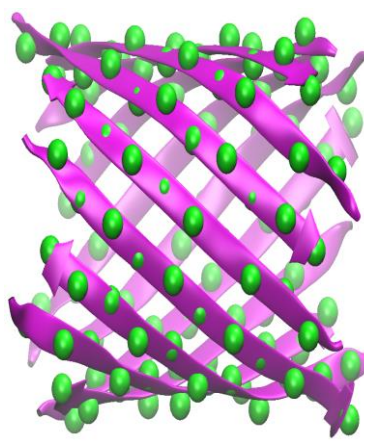
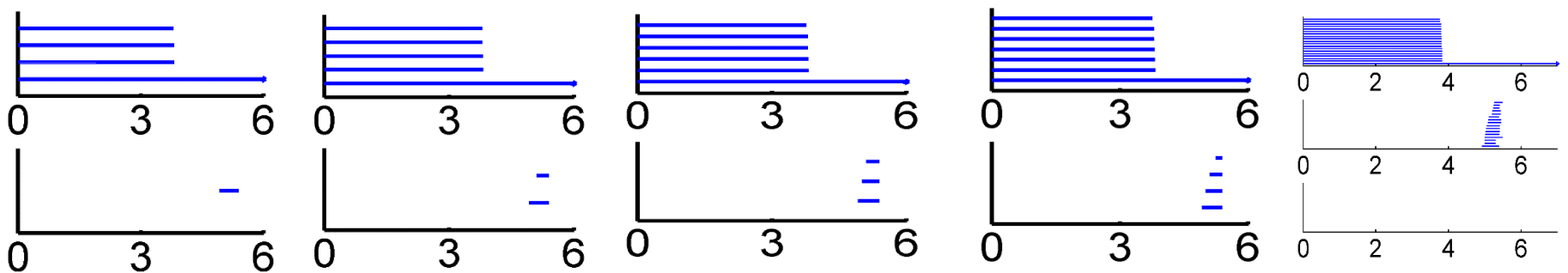
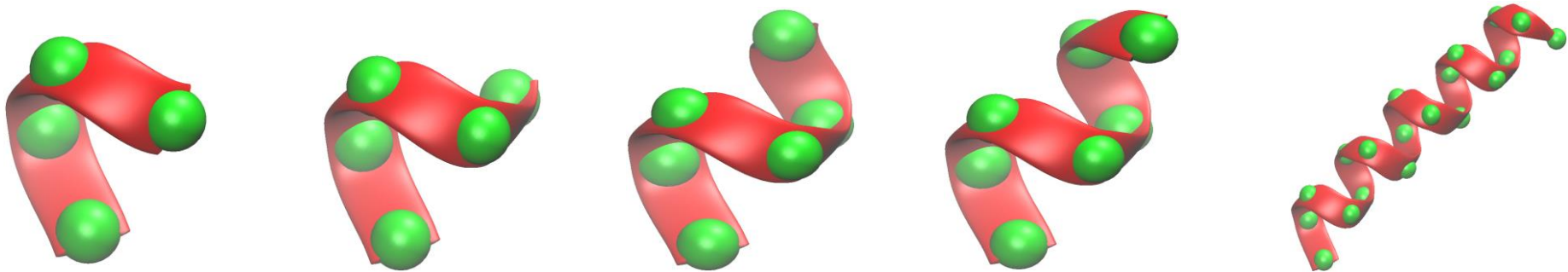
Algebraic Topology

Vietoris-Rips complexes, **persistent homology** and **topological fingerprint**

(Xia, Wei, 2014)



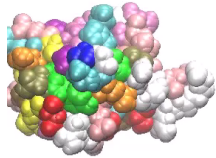
Topological fingerprints of an alpha helix



**(Xia & Wei,
IJNMBE,
2014, 2015)**

Algebraic Topology

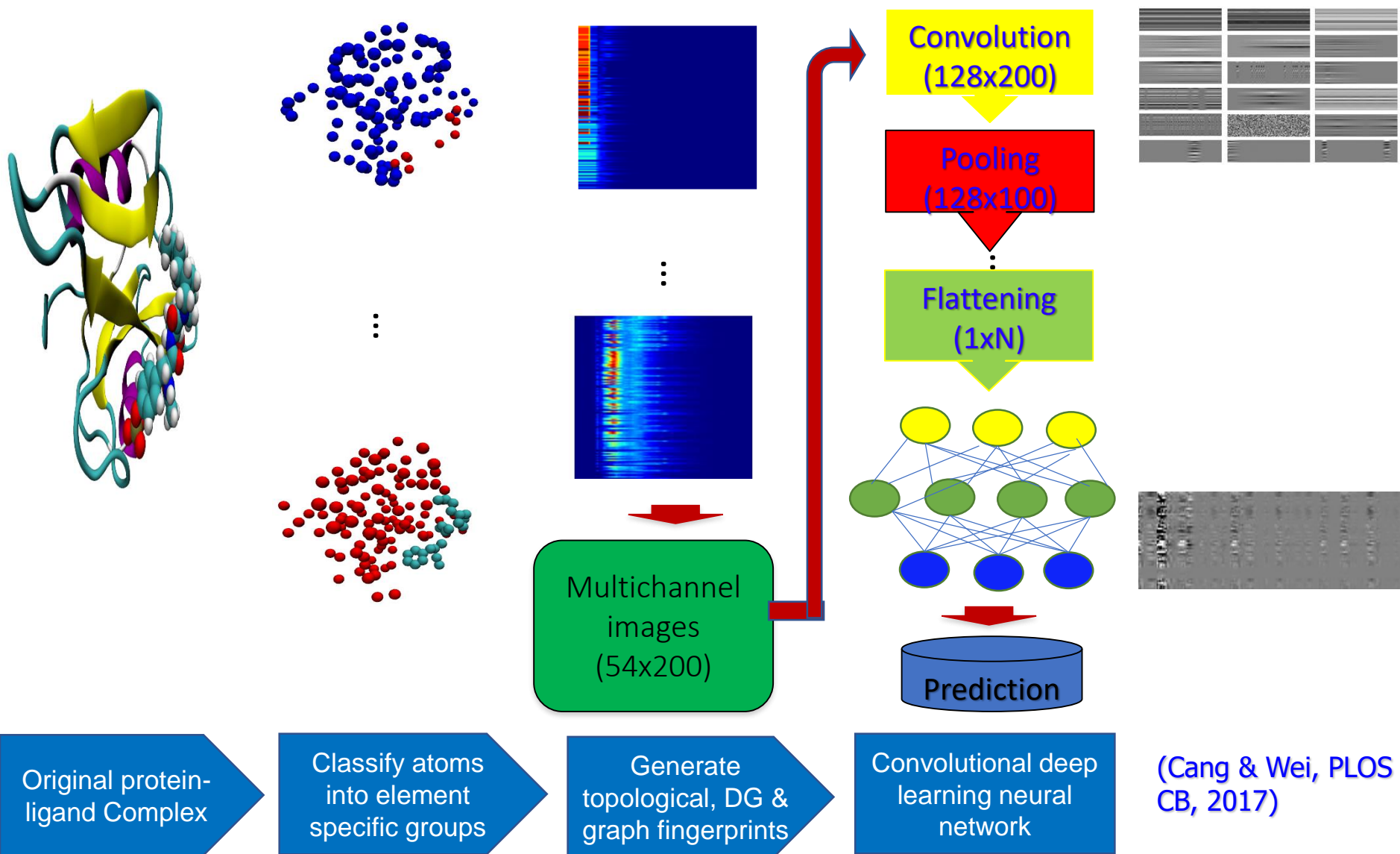
2D persistent homology of protein 1UBQ unfolding

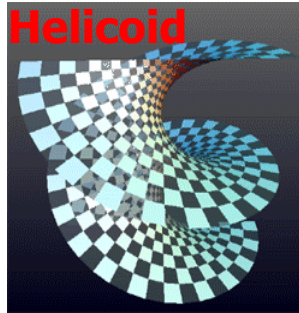
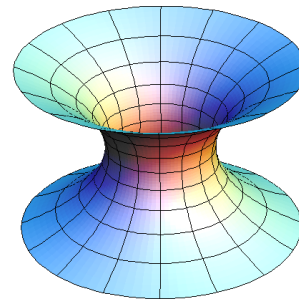
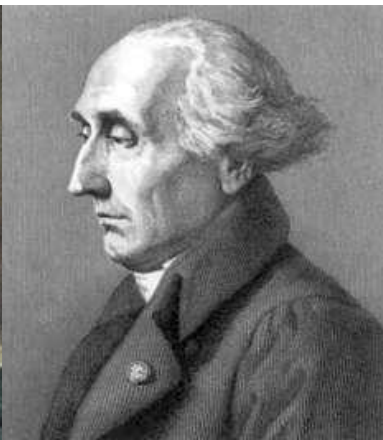


Kelin Xia

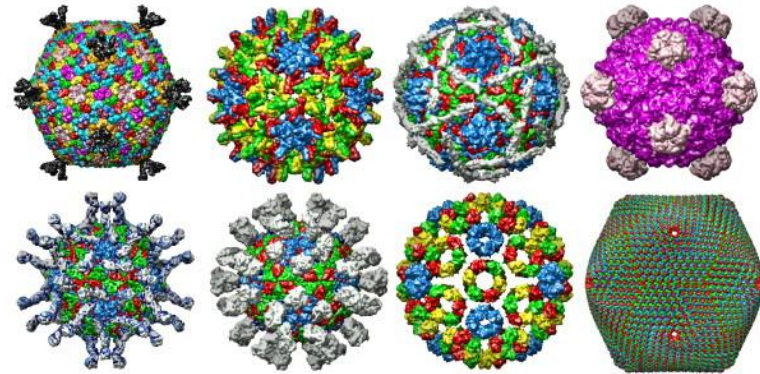
(Xia & Wei, JCC, 2015)

Topological convolutional deep Learning architecture





Helicoid



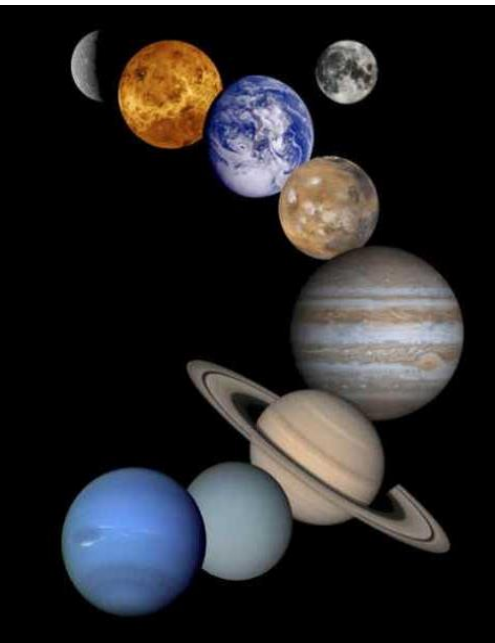
Viral morphology

Leonhard P. Euler
(Swiss Mathematician,
April 15, 1707 – Sept
18 1783)

Joseph L. Lagrange
(Italian Mathematician,
January 25 1736 –
April 10, 1813)

Minimal Surfaces

A way to minimize energy
and maximize stability



Man-made life,
Mycoplasma
mycoides

Differential geometry based minimal surface model

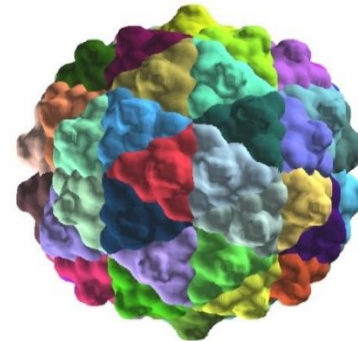
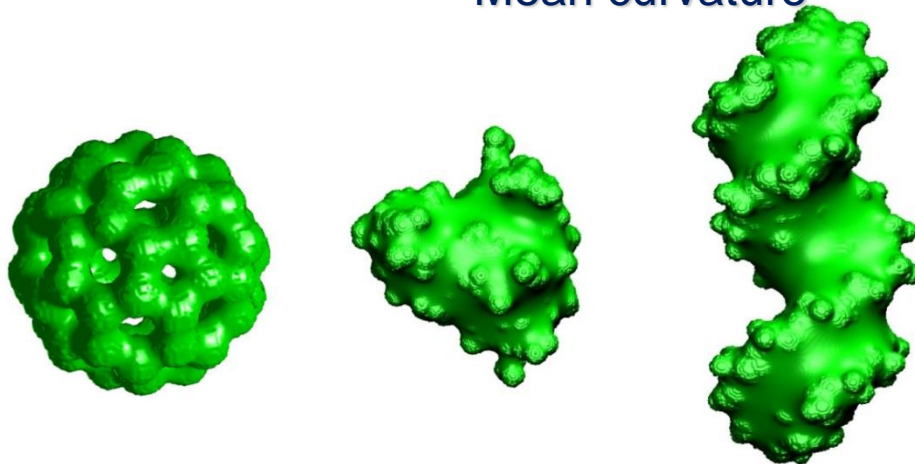
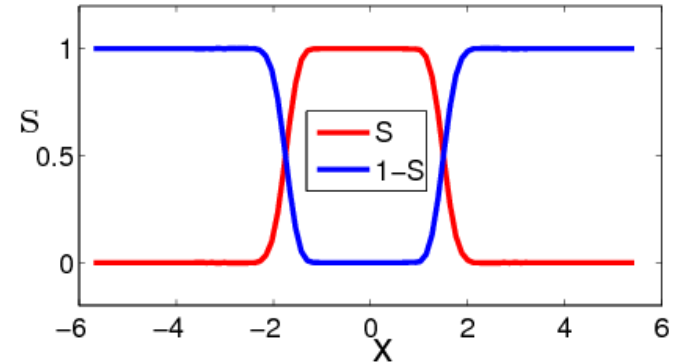
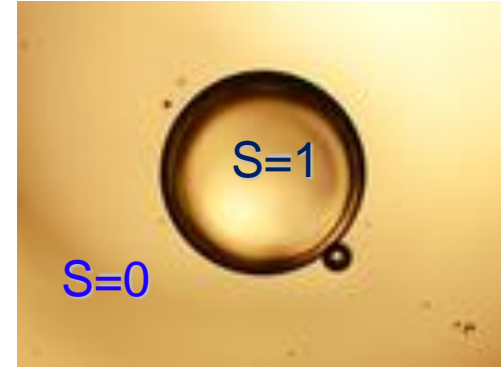
$$G = \int \gamma[\text{area}] dr \quad \text{area} = |\nabla S|$$

where G is the surface energy, γ is the surface tension, and S is a surface characteristic function:

Generalized Laplace-Beltrami flow:

$$\frac{\partial S}{\partial t} = |\nabla S| \left[\nabla \cdot \frac{\gamma \nabla S}{|\nabla S|} \right]$$

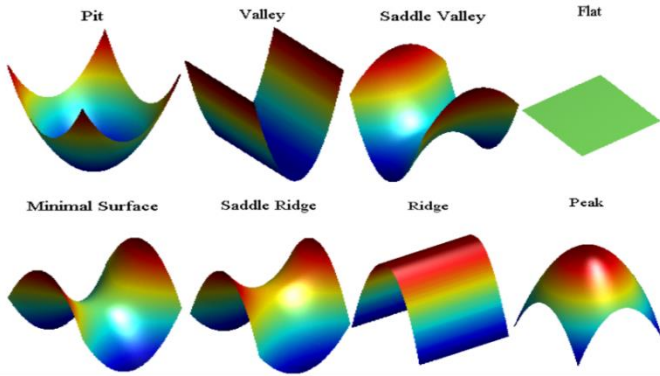
Mean curvature



Shan Zhao

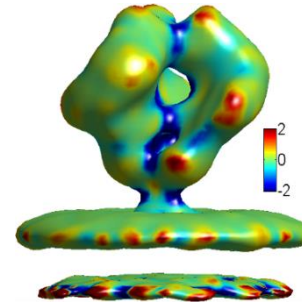
(Bates, Wei, Zhao, 2006; JCC,2008; Zhao, Cang, Tong & Wei, Bioinformatics 2018)

Differential Geometry

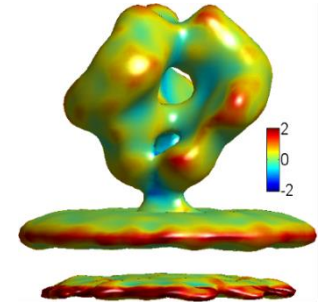


Kelin Xia

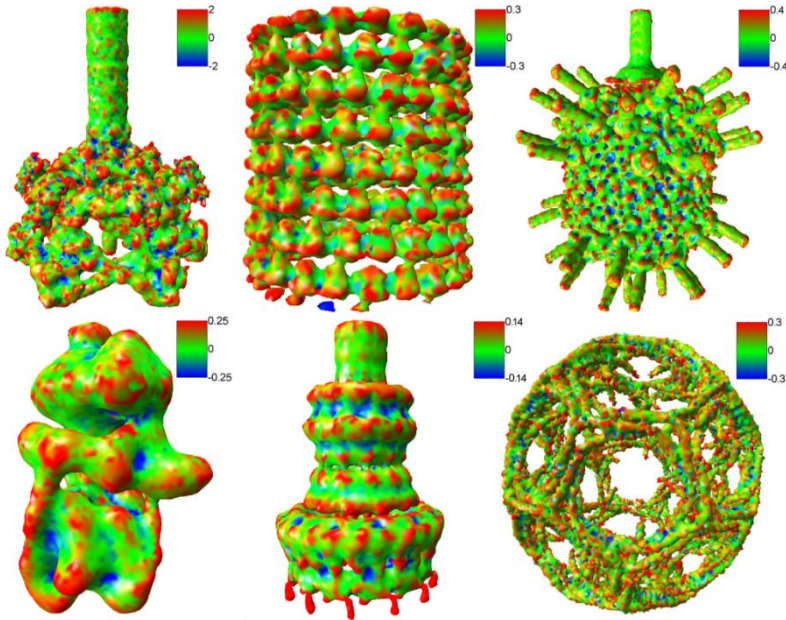
Gauss



Mean

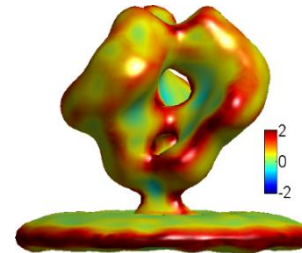


Mean curvatures of subcellular structures

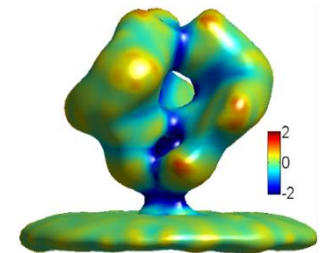


(Feng, Xia, Tong and Wei, JCP, IJNMBI, 2012)

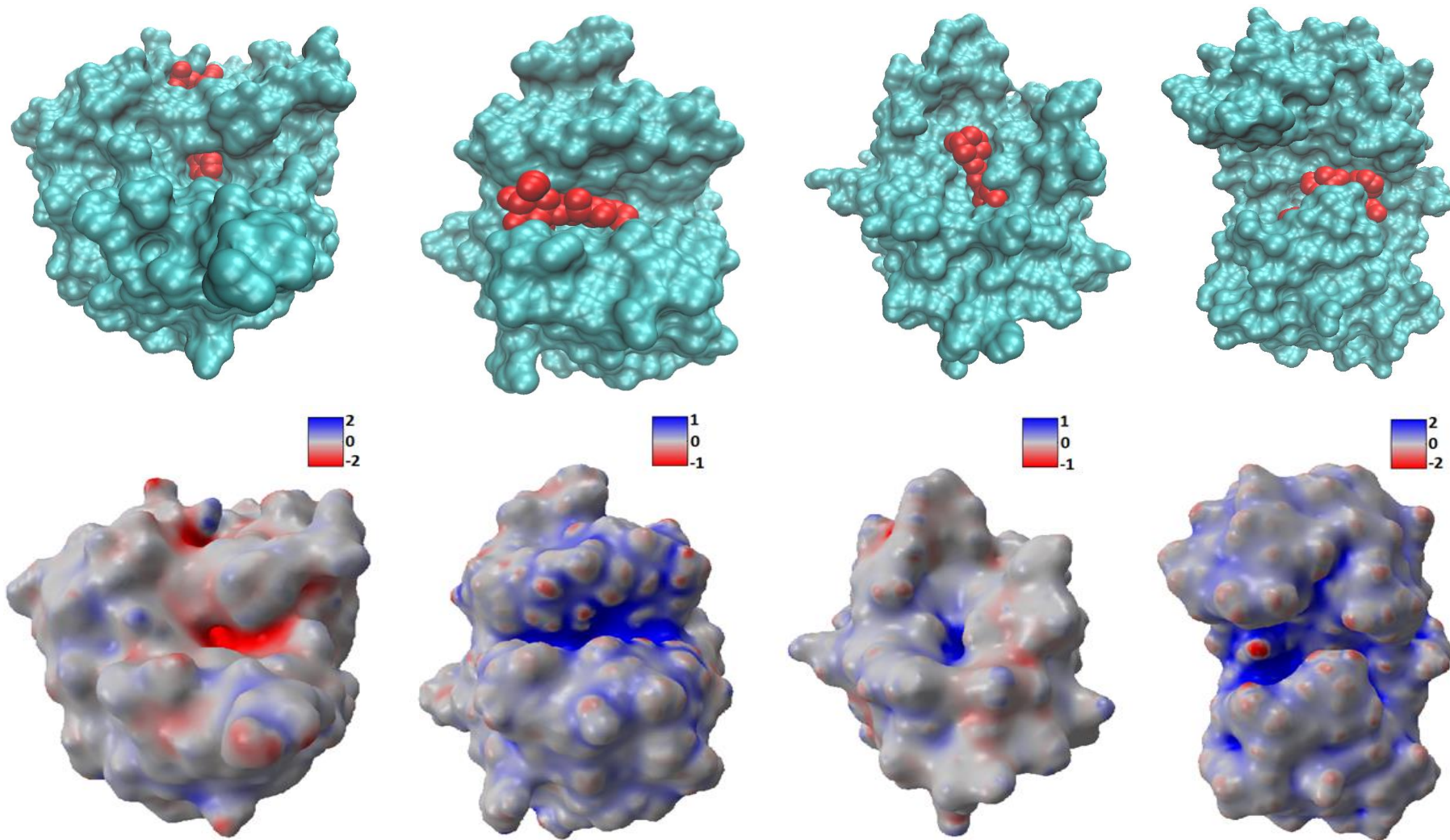
Minimum



Maximum



Protein binding site prediction by the product of curvature and electrostatics



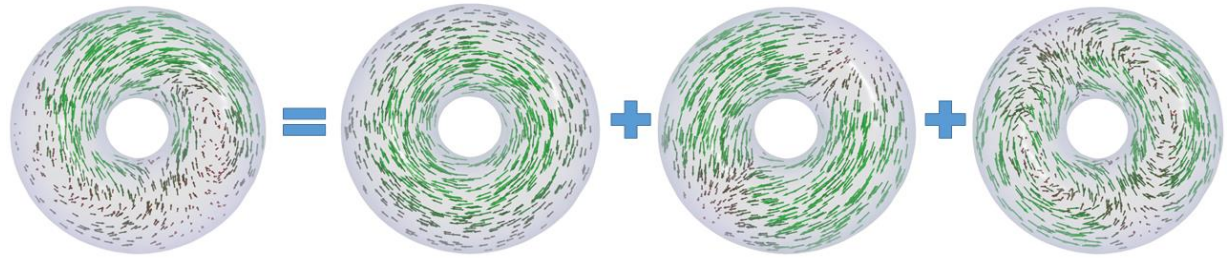
(Xia, Feng, Tong & Wei, JCP 2013; Zhao, Cang, Tong & Wei, Bioinformatics, 2018)

de Rham-Hodge theory and discrete exterior calculus

Hodge decomposition:

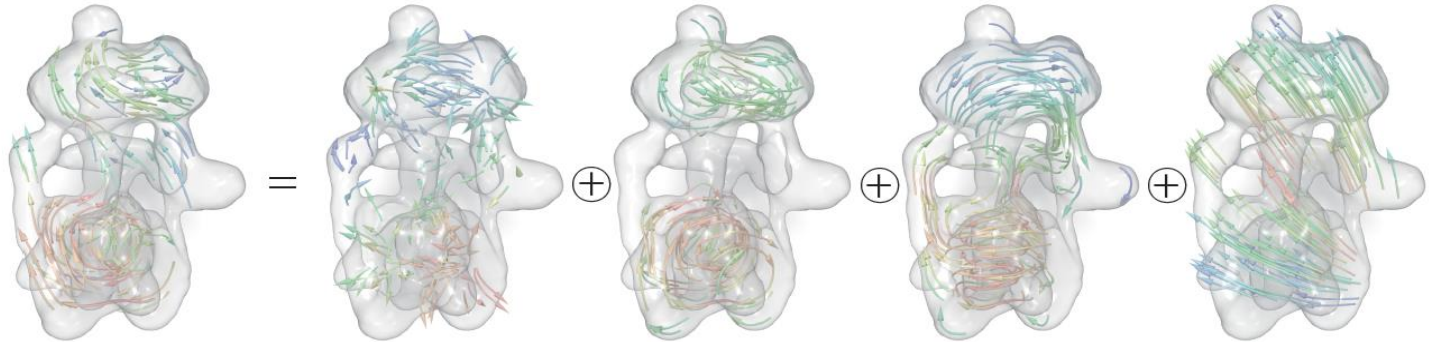


(Zhao, Wang,
Tong & Wei,
2018)



A vector field = Harmonic + curl-free + divergent-free

Cryo-EM data:



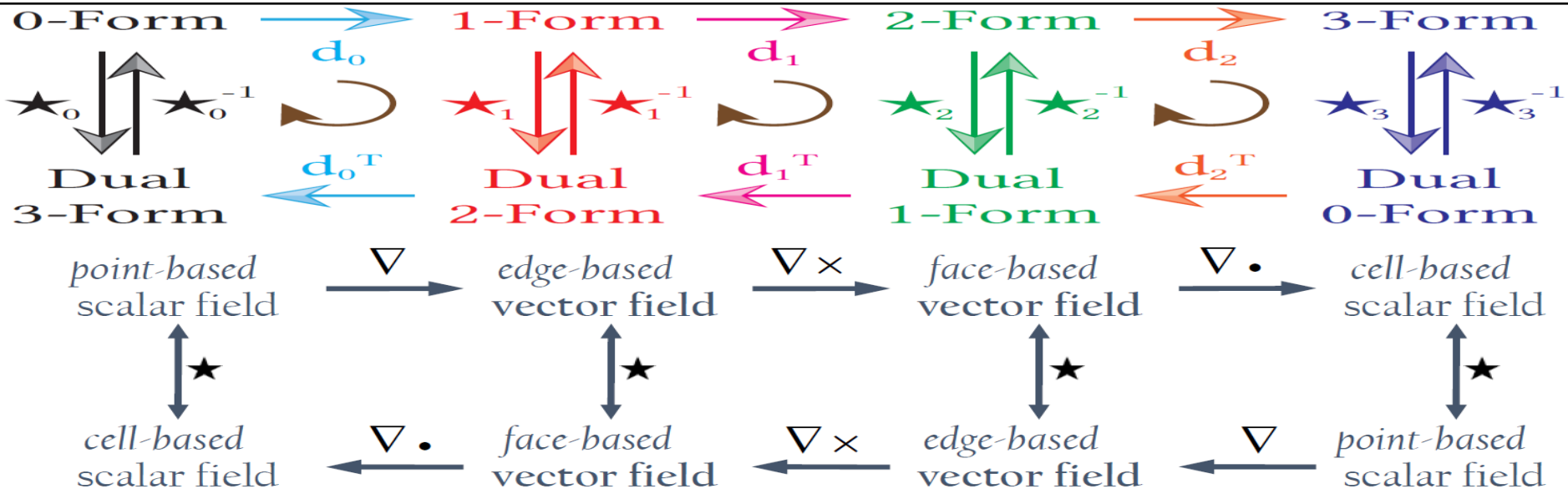
Input

Normal Gradient

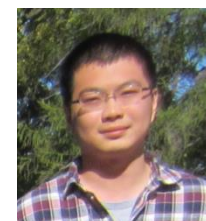
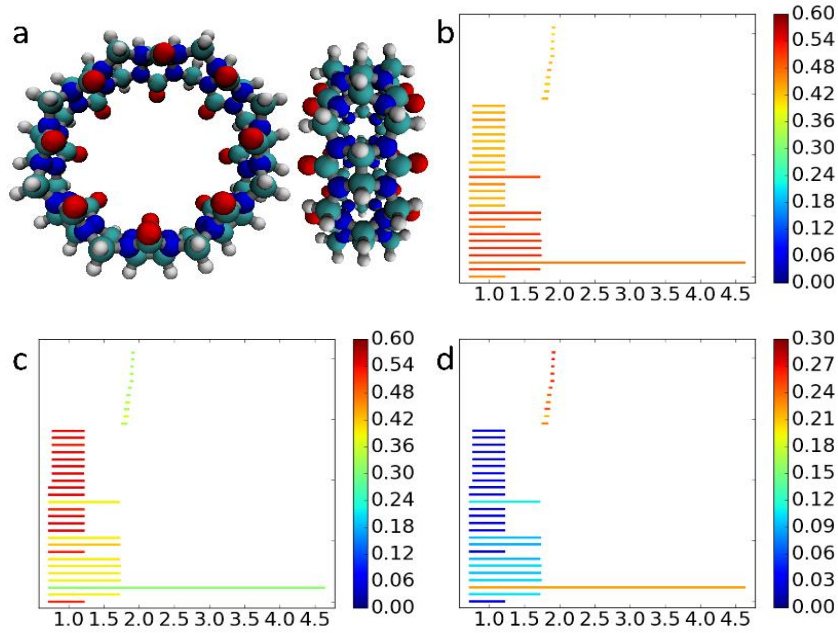
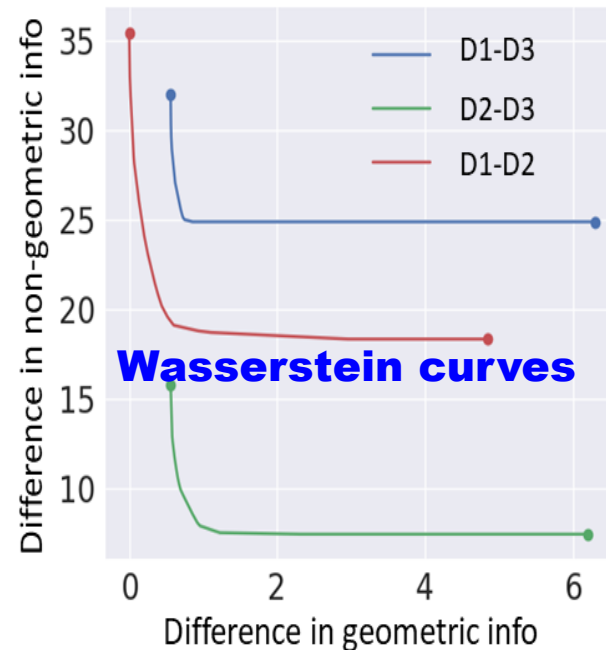
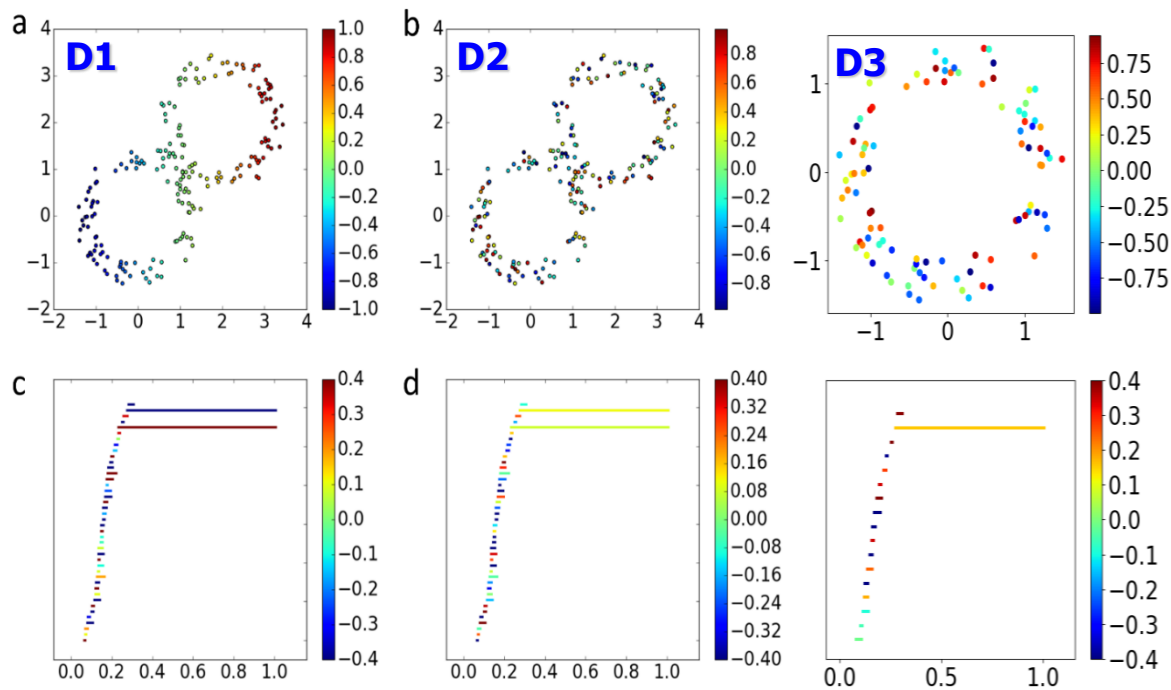
Tangential Curl

Tangential Harmonic

Central Harmonic



Persistent cohomology

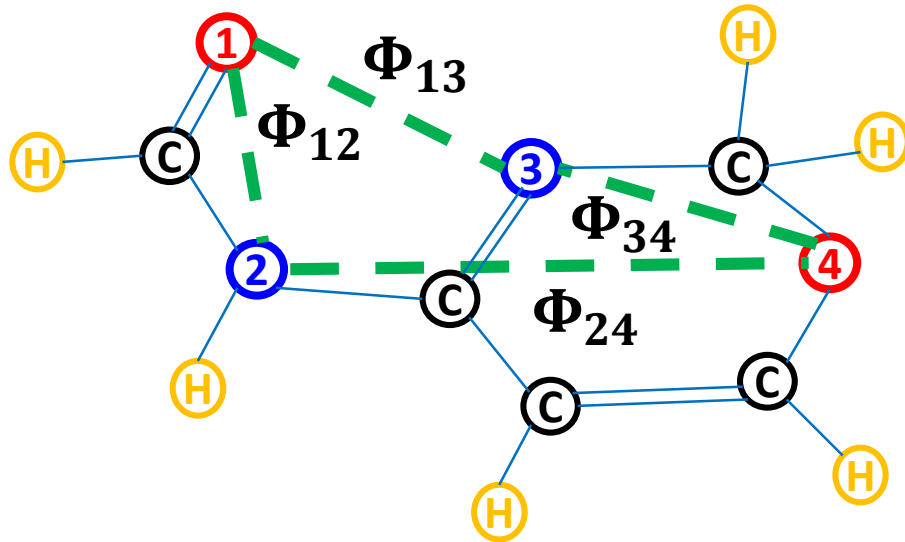


Zixuan Cang

(Cang & Wei, 2018)

Algebraic Graph Theory for Biomolecules

Molecular graph $G(V,E)$



Adjacency matrix
of $G(V_{ON},E)$

$$\begin{pmatrix} 0 & \Phi_{12} & \Phi_{13} & 0 \\ \Phi_{12} & 0 & 0 & \Phi_{24} \\ \Phi_{13} & 0 & 0 & \Phi_{34} \\ 0 & \Phi_{24} & \Phi_{34} & 0 \end{pmatrix}$$

Eigenvalues: $\lambda_1^A, \lambda_2^A, \dots$

Laplacian matrix of $G(V_{ON},E)$

$$\begin{pmatrix} \Phi_{12} + \Phi_{13} & -\Phi_{12} & -\Phi_{13} & 0 \\ -\Phi_{12} & \Phi_{12} + \Phi_{24} & 0 & -\Phi_{24} \\ -\Phi_{13} & 0 & \Phi_{13} + \Phi_{34} & -\Phi_{34} \\ 0 & -\Phi_{24} & -\Phi_{34} & \Phi_{24} + \Phi_{34} \end{pmatrix}$$

Eigenvalues: $\lambda_1^L, \lambda_2^L, \dots$

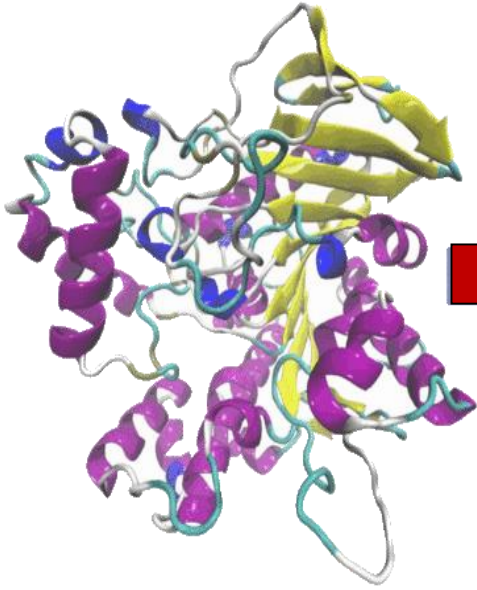
Can one hear the
shape of a drum?



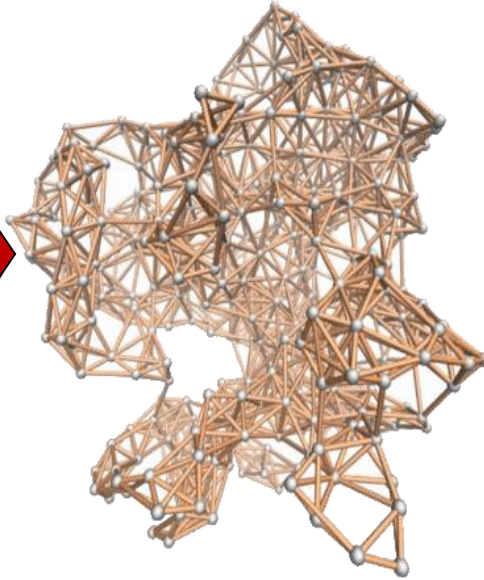
(Nguyen and Wei, 2018)

Algebraic graph theory for biomolecules

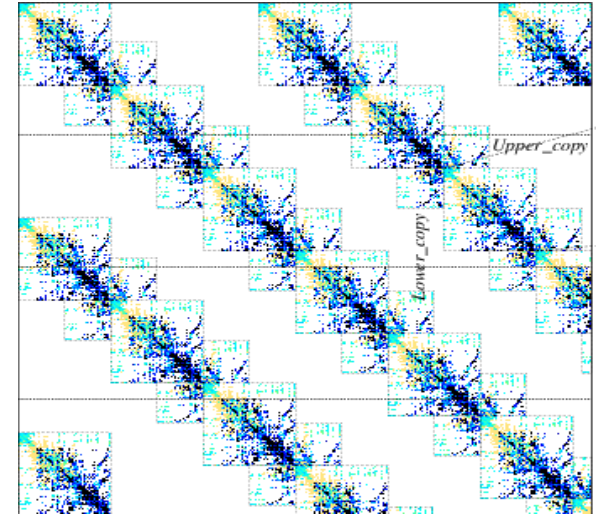
Protein



Hypergraph
representation



Laplacian matrices and
adjacency matrices



Eigenvalue multiplicities in Laplacian and adjacency matrices are associated with structural self-similarity, stability, flexibility and activity and hotspots, etc.

Corresponding eigenvalues $\lambda_1^L, \lambda_2^L, \dots$
 $\lambda_1^A, \lambda_2^A, \dots$

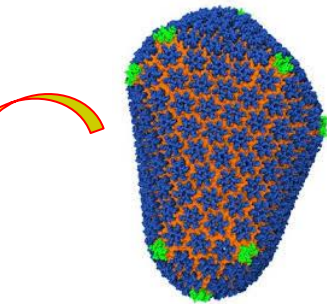
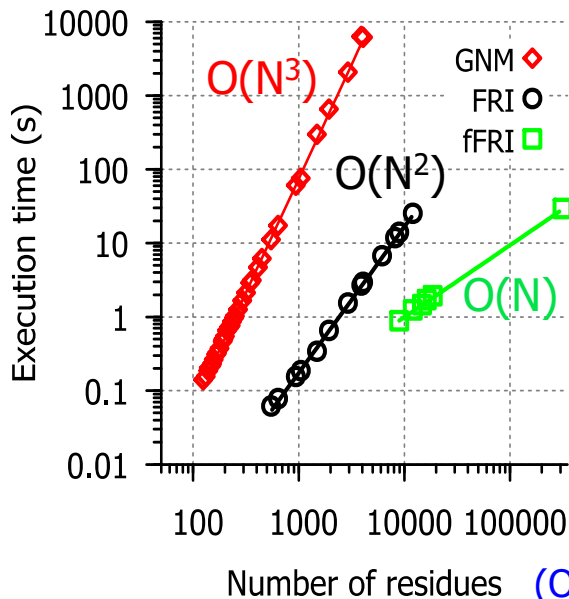
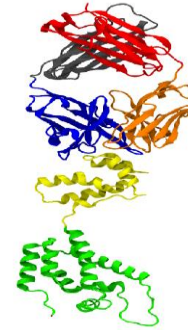
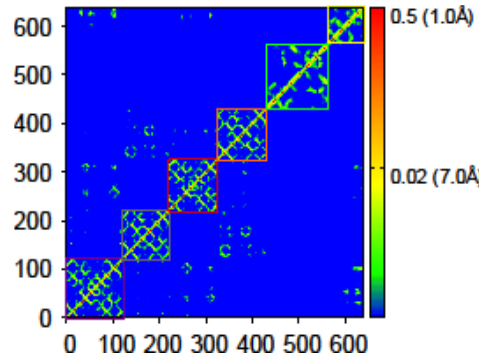
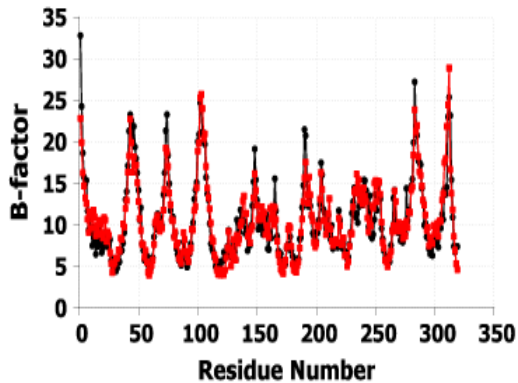
Mark Kac: Can one hear the shape of a drum?
Can one hear the interaction of molecules?

Geometric Graph Theory



K. Opron

- **Multiscale weighted colored graphs (MWCG)**
- MWCG is about 40% more accurate than Gaussian network model (GNM) in B-factor prediction, based on 364 proteins.



HIV capsid (313,236 residues) would takes GNM 120 years to compute!

$$\Gamma_{ij}(\Phi) = \begin{cases} -\Phi(r_{ij}, \eta), & i \neq j, \\ -\sum_{j, j \neq i}^N \Gamma_{ij}, & i = j \end{cases}$$

$$\Phi(r_{ij}, \eta) = 1, \quad r_{ij} \rightarrow 0$$

$$\Phi(r_{ij}, \eta) = 0 \quad r_{ij} \rightarrow \infty$$

$$\Phi(r_{ij}, \eta) = e^{-(r_{ij}/\eta)^\kappa}$$

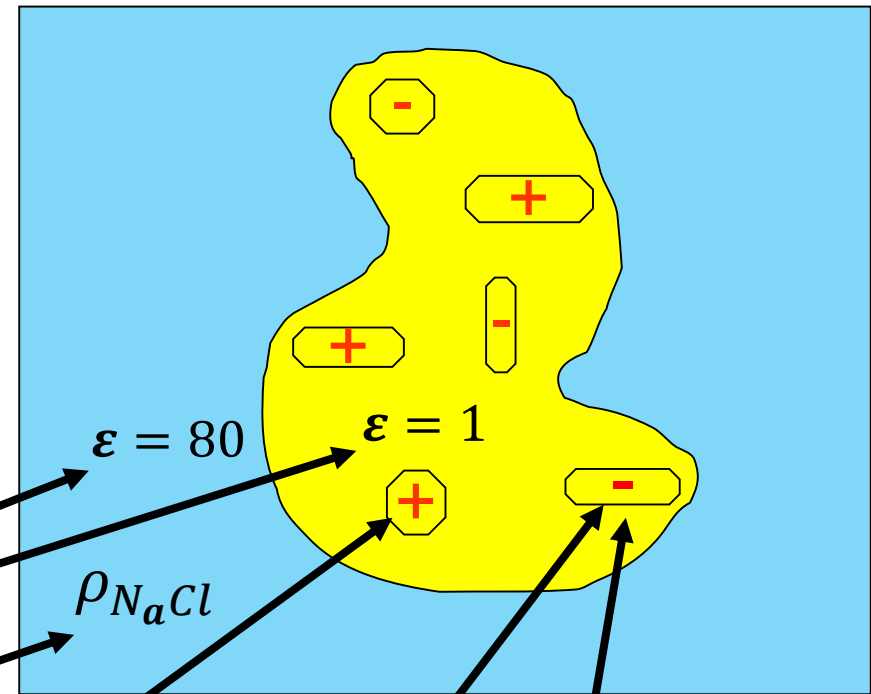
$$B_i^{FRI} = a(\Gamma_{ii}(\Phi))^{-1}$$

(Opron, Xia and Wei, JCP, 2013; JCP 2014; JCP, 2015; Nguyen, et al, JCI, 2017, Bramer and Wei, JCP, 2018. Nguyen and Wei, 2018)

Multiscale: The Poisson-Boltzmann equation

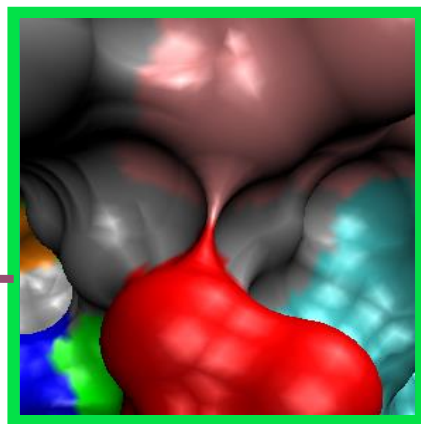
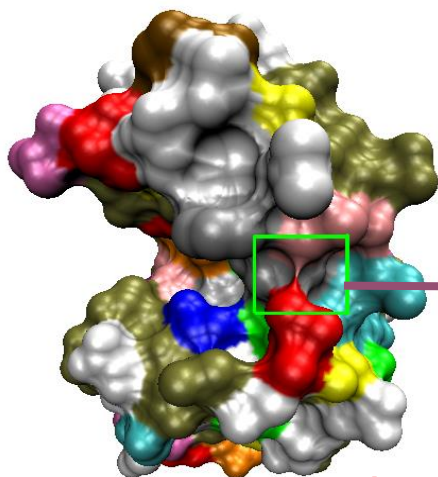
- Discontinuous dielectric constant at the interface
- Non-smooth interface (geometric singularity)
- Singular charges (delta functions)

Chern et al, 2003; Geng, Yu, Wei, JCP, 2007; Geng, Zhao, JCP 2017

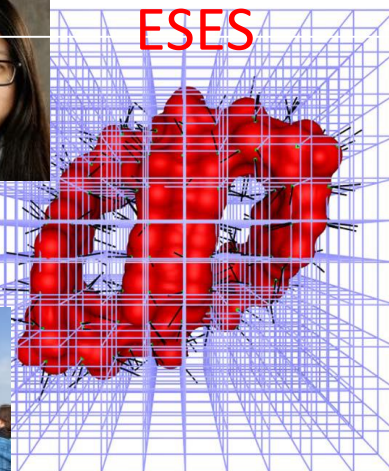
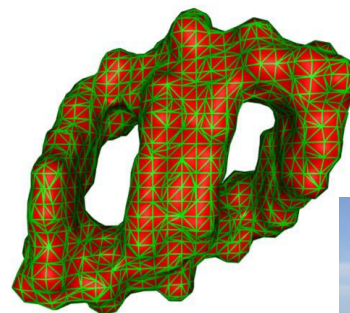


$$-\nabla \cdot (\boldsymbol{\varepsilon}(\mathbf{r})\nabla\phi) = \sum_i q_i c_i e^{-\frac{q_i\phi}{kT}} + \sum_i \underbrace{(Q_i\delta(\mathbf{r} - \mathbf{r}_i))}_{\text{Point charge}} - \underbrace{d_i \cdot \nabla\delta(\mathbf{r} - \mathbf{r}_i) + \Theta_i : \nabla\nabla\delta(\mathbf{r} - \mathbf{r}_i)}_{\text{Charge polarization (Amoeba)}}$$

MIBPB for solving the Poisson equation with protein interface

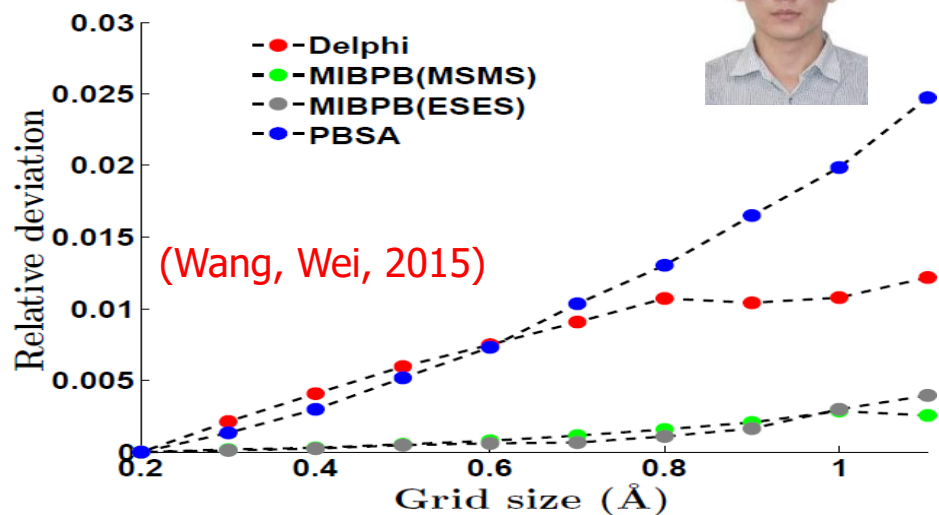


(Yu, Geng, Wei, JCP 2007)



(Liu, Wang, Zhao, Tong, Wei, JCC 2017)

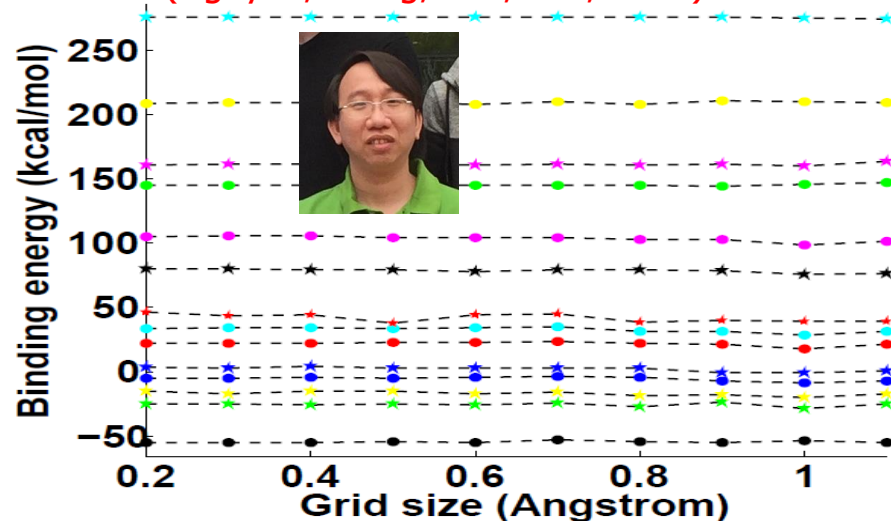
Relative solvation energy deviations over grid refinement for 947 proteins in the Amber test set



(Wang, Wei, 2015)

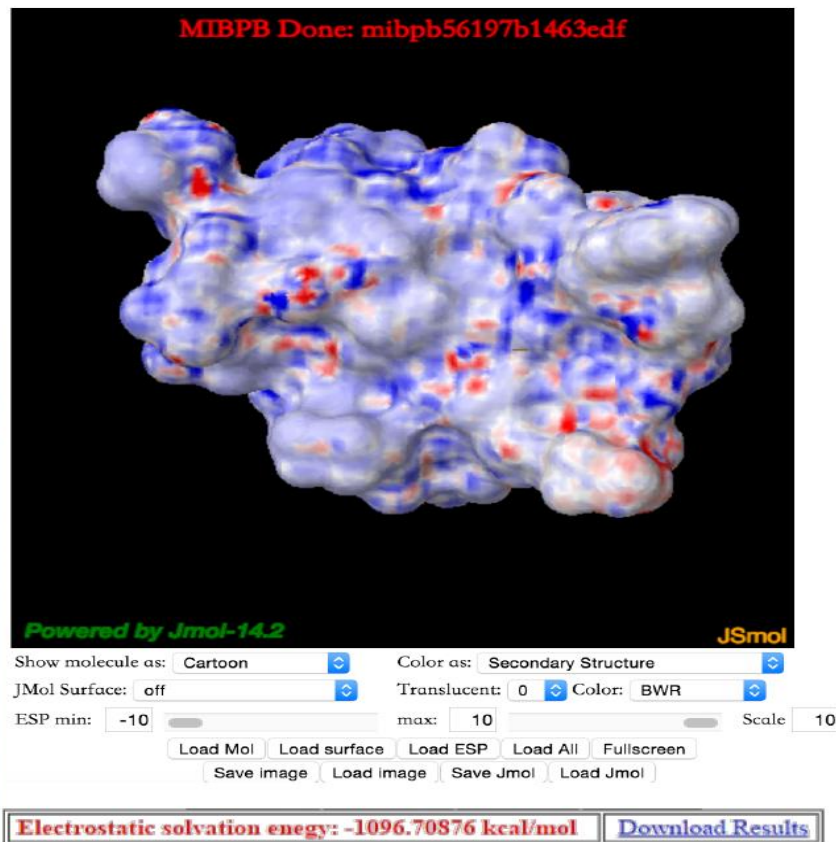
Electrostatic binding energies of 14 RNA-protein complexes over grid refinement

(Nguyen, Wang, Wei, JCC, 2015)



MIBPB is a software package for obtaining electrostatic potential and solvation free energy via solving the Poisson-Boltzmann (PB) equation. It makes use of the second order convergent MIB technique and is essentially grid independent. Its mean relative error is less than 0.5% for about 1000 test proteins when the grid size is refined from 1.1 to 0.2 Angstrom. [\[User Manual\]](#)

Input_File:	<input checked="" type="radio"/> PDB ID: <input type="text"/> Chains: * <input type="radio"/> User File: <input type="button" value="选择文件"/> 未选择任何文件 *
MIBPB Options	1.0 Interior Dielectric
	80 Exterior Dielectric
	0.8 Grid Resolution
	0.0 Ion Strength
	1.4 Surface Probe Radius for MIBPB
	Linearized PB: <input checked="" type="checkbox"/> Yes Simplified Solver: <input type="checkbox"/> Yes
Resulting Surface Options <input checked="" type="checkbox"/> Yes	1.4 Probe Radius
	0.8 Grid Resolution
	2.0 Grid Extension
PDB2PQR Options <input checked="" type="checkbox"/> Yes	Force Field: AMBER
	Protonation: at pH: 7.0 by: PROPKA
	Remove Water: <input checked="" type="checkbox"/> Yes
	Remove Hydrogen: <input type="checkbox"/> Yes
Small molecule to PQR Options <input type="checkbox"/> Yes	Only assign charges and radius: <input type="checkbox"/> Yes
	Charge Type: AM1-BCC Radius Type: mbondi
pKa Calculation <input type="checkbox"/> Yes	Residue Type: ASP Residue ID: <input type="text"/> <input type="button" value="Analyze online PDB"/>
	Job Title: <input type="text"/>
User Email: <input type="text"/>	
<input type="button" value="Default"/> <input type="button" value="Submit"/> <input type="button" value="Clear Job"/>	



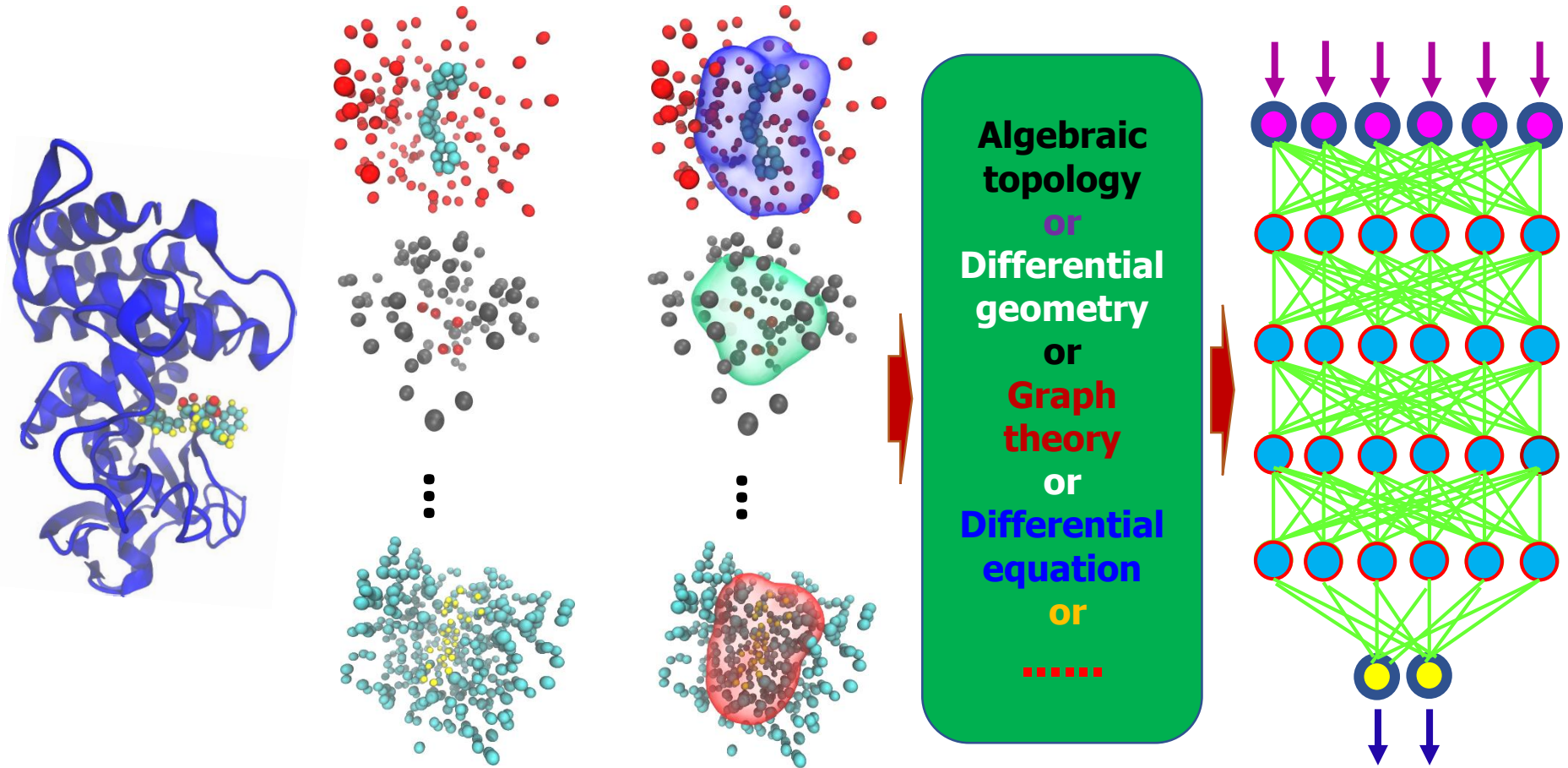
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- For industrial/commercial users, a moderate license fee may apply. Please contact us directly at wei@math.msu.edu.
- If you have any questions or bugs to report, please feel free to contact: wangbaonj@gmail.com

(Wang, Zhao, Wei, 2015)



Mathematical deep learning



Protein-ligand complex

Element specific groups

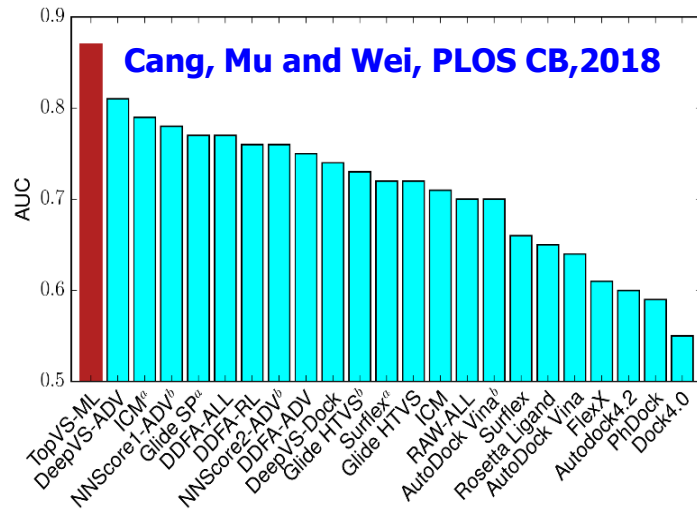
Element interactive manifolds

Various Mathematical features

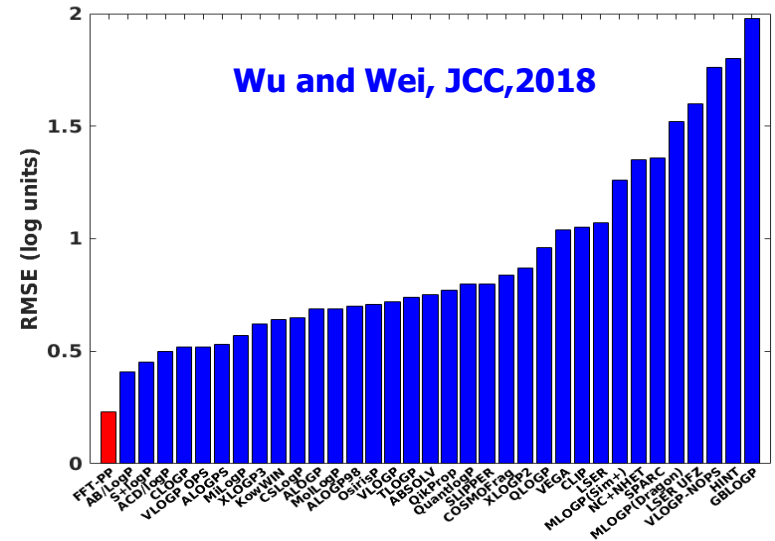
Machine learning prediction

Topological learning based predictions

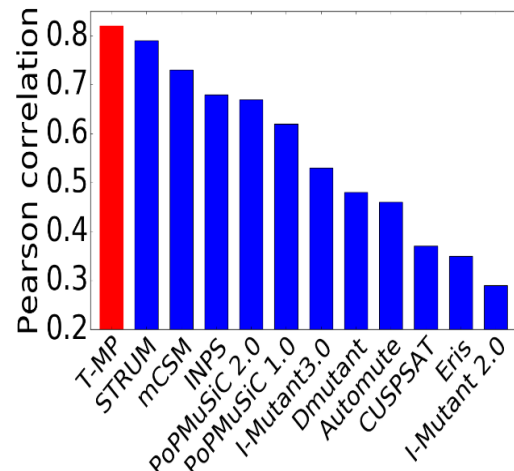
Classification of ligands & decoys DUD database 128,374 protein- ligand/decoy pairs



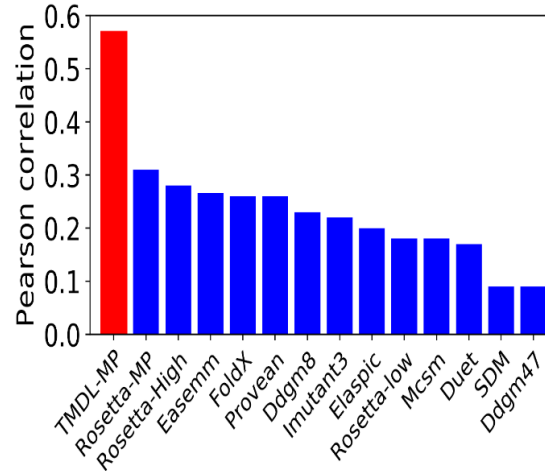
Prediction RMSD of LogP (Star set)



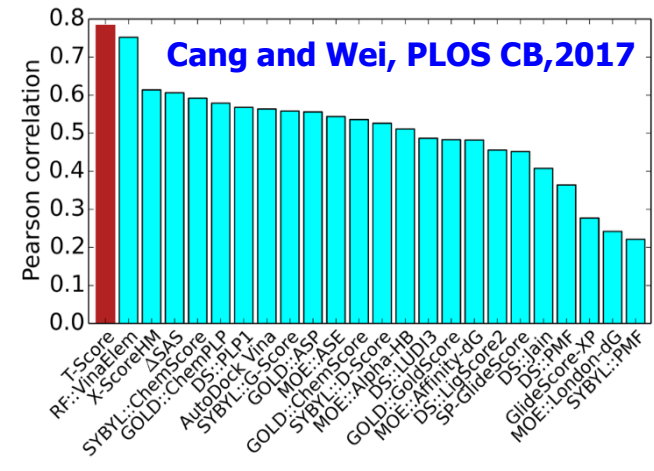
Predicting mutations on 2648 globular proteins (Cang and Wei, Bioinformatics, 2017)



Predicting mutations on 223 membrane proteins



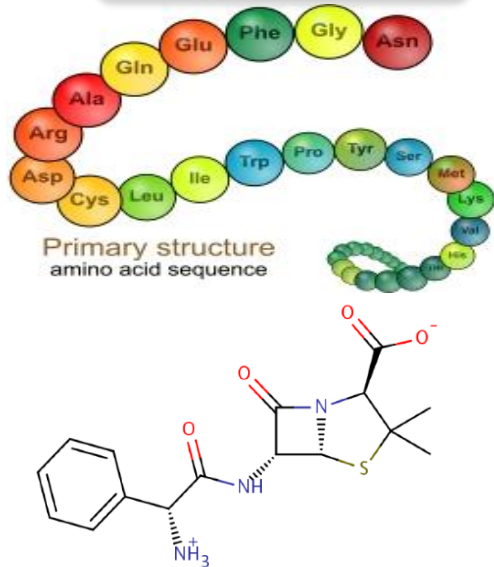
Binding affinity prediction of PDBBind v2013 core set of 195 protein-ligand complexes



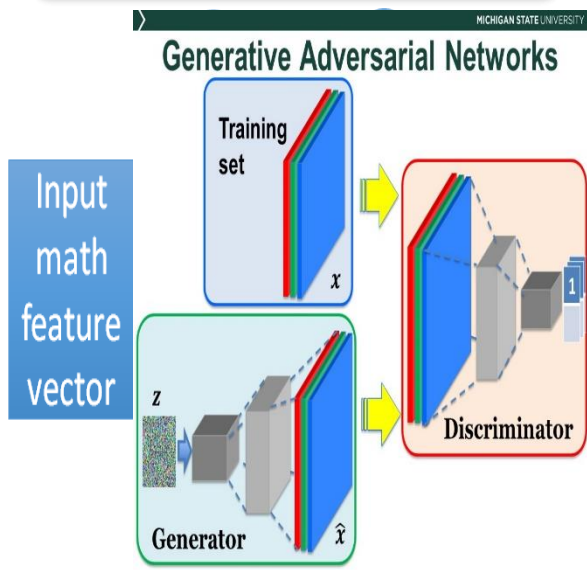


Drug Design Data Resource (D3R) Grand Challenge

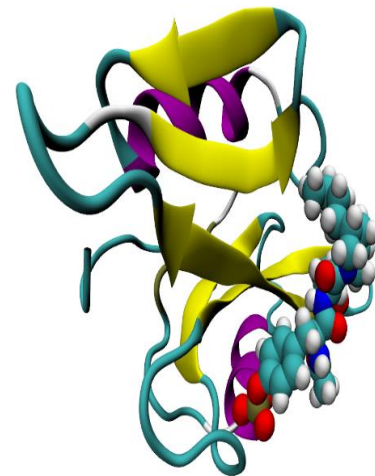
Given data



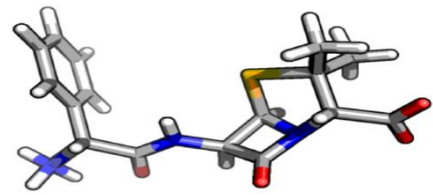
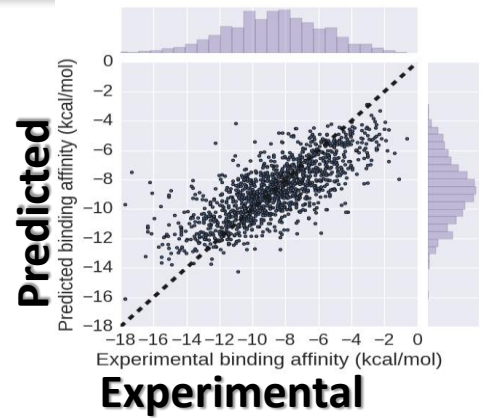
Math based GAN



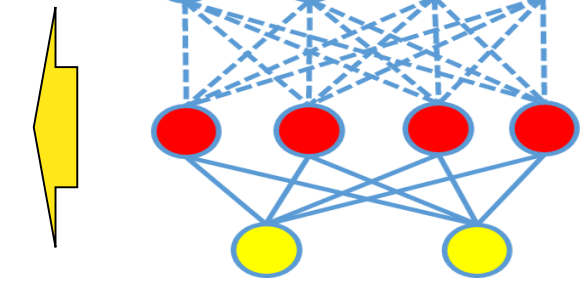
Predicted complex



Final predictions to be compared with experiments

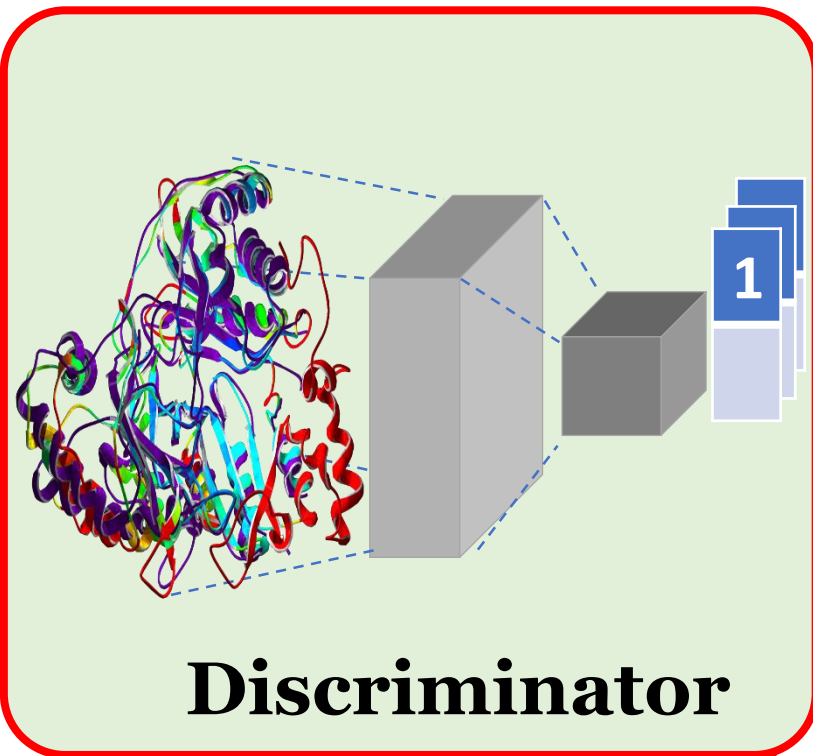
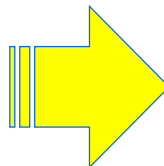
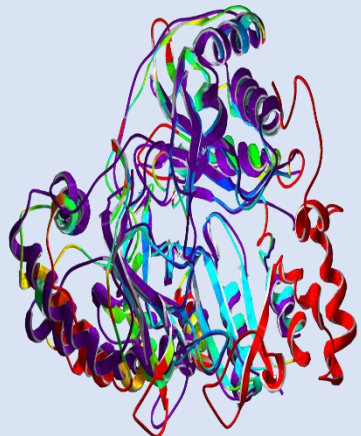


Drug pose

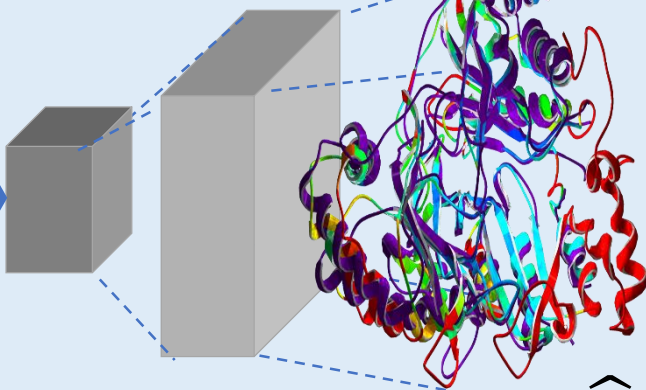


Generative Adversarial Networks for Drug Design

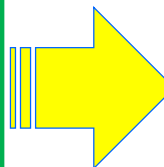
Training set



z



\hat{x}



D3R Grand Challenge 2 (2016-2017)



Given: Farnesoid X receptor (FXR) and 102 ligands

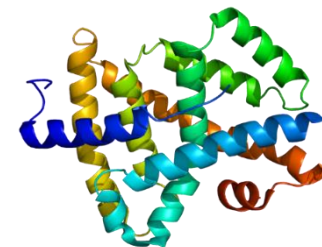
Tasks: Dock 102 ligands to FXR, and predict their poses, binding free energies and energy ranking

Stage 1

- [Pose Predictions \(partials\)](#)
- [Scoring \(partials\)](#)
- [Free Energy Set 1 \(partials\)](#)
- [Free Energy Set 2 \(partials\)](#)

Stage 2

- [Scoring \(partials\)](#)
- [Free Energy Set 1 \(partials\)](#)
- [Free Energy Set 2 \(partials\)](#)

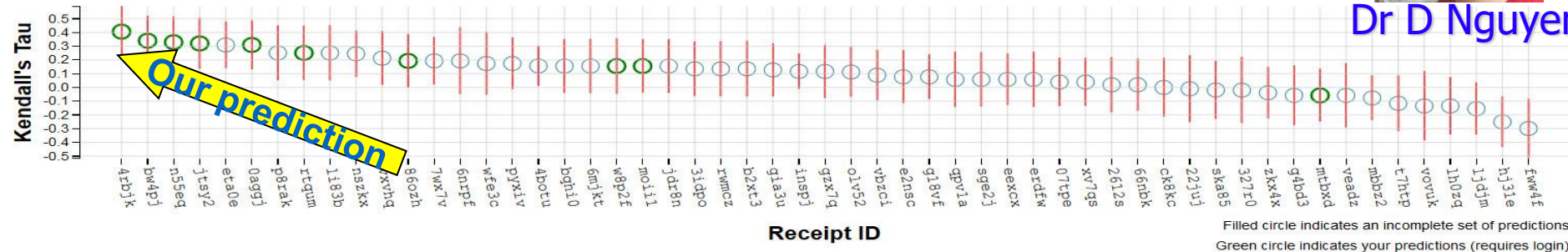


Dr D Nguyen

Grand Challenge 2

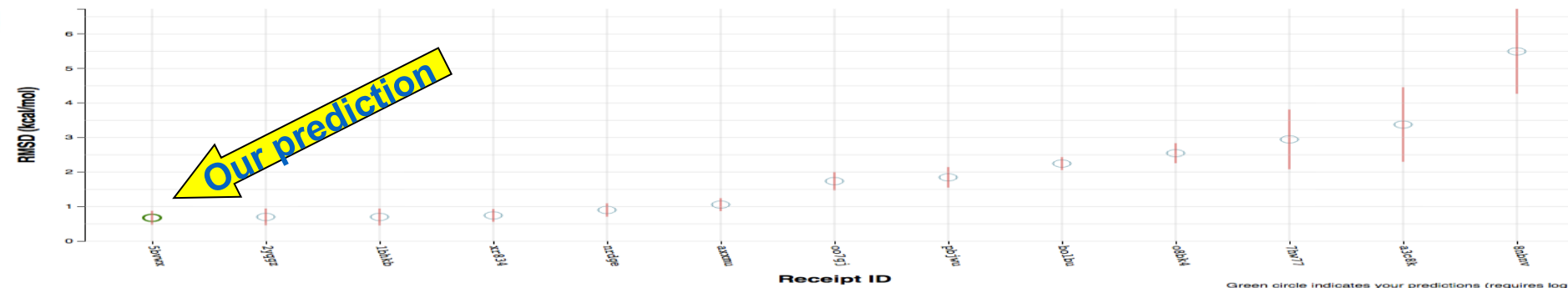
(Nguyen et al, JCAMD, 2018)

Free Energy Set 1 (Stage 2) - Kendall's Tau



Grand Challenge 2

Free Energy Set 1 (Stage 1) - RMSD



D3R Grand Challenge 3 (2017-2018)

(Nguyen et al, JCAMD, 2018)



Pose Prediction

Cathepsin Stage 1A

[Pose Predictions \(partials\)](#)

Affinity Rankings excluding Kds > 10 μM

Cathepsin Stage 1

[Scoring \(partials\)](#)

[Free Energy Set](#)

VEGFR2

[Scoring \(partials\)](#)

JAK2 SC3

[Scoring](#)

[Free Energy Set](#)



Active / Inactive Classification

VEGFR2

[Scoring \(partials\)](#)

JAK2 SC3

[Scoring](#)

[Free Energy Set](#)



Affinity Rankings for Cocrystalized Ligands

Cathepsin Stage 1

[Scoring \(partials\)](#)

[Free Energy Set](#)



Cathepsin Stage 1B

[Pose Prediction](#)

Cathepsin Stage 2

[Scoring \(partials\)](#)

[Free Energy Set](#)

JAK2 SC2

[Scoring \(partials\)](#)

TIE2

[Scoring](#)

[Free Energy Set 2](#)



JAK2 SC2

[Scoring \(partials\)](#)

TIE2

[Scoring \(partials\)](#)

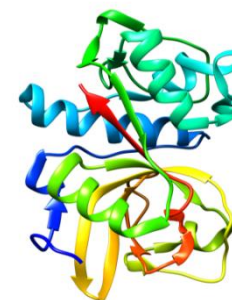
[Free Energy Set 1](#)



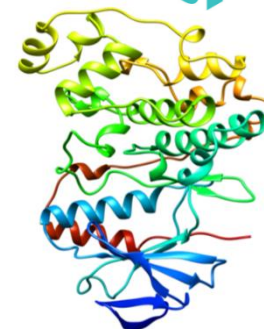
Cathepsin Stage 2

[Scoring \(partials\)](#)

[Free Energy Set](#)



Cathepsin S



Kinase: p38-α

p38-α

[Scoring](#)

ABL1

[Scoring \(partials\)](#)



p38-α

[Scoring \(partials\)](#)

ABL1

[Scoring \(partials\)](#)



Zixuan Cang



Dr D Nguyen

D3R Grand Challenge 4 (2018-2019)



Pose Predictions

BACE Stage 1A
Pose Predictions (Partials)

BACE Stage 1B
Pose Prediction (Partials)

Affinity Predictions

Cathepsin Stage 1
Combined Ligand and Structure Based Scoring

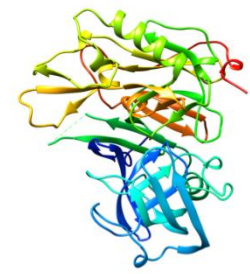


Dr. Kaifu Gao Dr. D Nguyen

Ligand Based Scoring (No participation)

Structure Based Scoring

Free Energy Set



BACE Stage 1
Combined Ligand and Structure (No participation)

Ligand Based Scoring (Partials) (No participation)

Structure Based Scoring (Partials) (No participation)

Free Energy Set (No participation)

BACE Stage 2
Combined Ligand and Structure

Ligand Based Scoring (No participation)

Structure Based Scoring (Partials)

Free Energy Set

Algebraic topology

Differential topology

Geometric topology

Differential equation

Number theory

Biology became microscopic (i.e., molecular) in 1960s and added an omics dimension around the dawn of the millennium.

Algebraic geometry

Algebraic graph

Differential geometry

Structural graph

Distance geometry

Exyreme graph

Driving by mathematics, biology is transforming from qualitative, phenomenological and descriptive to quantitative, predictive and analytical.

The last frontier of science is biology, while the last frontier of biology is mathematics.

Lie algebra

Geometric algebra

Complex analysis

Real analysis

Stochastic analysis

