1. Consider $\left|3x+1-10\right| < \epsilon \Rightarrow \left|3x-9\right| < \epsilon \Rightarrow \left|3(x-3)\right| < \epsilon$

$\Rightarrow |x-3| < \frac{\epsilon}{3}$, so δ can be $\frac{\epsilon}{3}$ or smaller and the largest value of δ is $\frac{\epsilon}{3}$.

2. $f(x) = \dfrac{x^3}{(x-1)(x-4)} + 6$

(1) definition domain: $x \neq 1, x \neq 4 \Rightarrow x \in (-\infty, 1) \cup (1,4) \cup (4, \infty)$.

moreover, for any $x_0 \in (-\infty, 1) \cup (1,4) \cup (4, \infty)$, $\lim_{x \to x_0} f(x) = f(x_0)$

So $f(x)$ is continuous in $(-\infty, 1) \cup (1,4) \cup (4, \infty)$

(2) $f(2) = \dfrac{2^3}{(2-1)(2-4)} + 6 = 2 > 0$

$f(3) = \dfrac{3^3}{(3-1)(3-4)} + 6 = \frac{27}{-2} + 6 = -\frac{15}{2} < 0$

$\Rightarrow 0 \in (f(3), f(2))$

and $f(x)$ is continuous in $[2, 3]$, so by the intermediate value theorem there exists an $x_0 \in (2,3)$ and $f(x_0) = 0$.

x_0 is the root of $f(x)$ in the interval $(1,4)$.