1. $f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{f(h) - \sqrt{0+1}}{h}$

$= \lim_{h\to 0} \frac{f(h) - 1}{h}$

$\lim_{h\to 0^+} \frac{f(h)-1}{h} = \lim_{h\to 0^+} \frac{\sqrt{h+1} - 1}{h} = \lim_{h\to 0^+} \frac{(\sqrt{h+1} - 1)(\sqrt{h+1} + 1)}{h \cdot (\sqrt{h+1} + 1)}$

$= \lim_{h\to 0^+} \frac{h}{h \cdot (\sqrt{h+1} + 1)} = \lim_{h\to 0^+} \frac{1}{\sqrt{h+1} + 1} = \frac{1}{2}$

$\lim_{h\to 0^-} \frac{f(h)-1}{h} = \lim_{h\to 0^-} \frac{(h^2+1) - 1}{h} = \lim_{h\to 0^-} h = 0$

$\frac{1}{2} \neq 0 \Rightarrow \lim_{h\to 0} \frac{f(h)-1}{h}$ DNE $\Rightarrow f'(0)$ DNE

2. $f'(x) = \left(\frac{x}{x+1}\right)' \cdot (\sqrt{x} + 1) + \frac{x}{x+1} \cdot (\sqrt{x} + 1)'$

$= \frac{x' \cdot (x+1) - x \cdot (x+1)'}{(x+1)^2} \cdot (\sqrt{x} + 1) + \frac{x}{x+1} \cdot \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{1}{(x+1)^2} \cdot (\sqrt{x} + 1) + \frac{1}{2} \frac{\sqrt{x}}{x+1} = \frac{\sqrt{x} + 1}{(x+1)^2} + \frac{1}{2} \frac{\sqrt{x}}{x+1}$