

Quiz 5 Solution

1. $y = y(x)$, $\frac{d}{dx}(x^3 + y^3 + xy) = \frac{d}{dx}(3)$

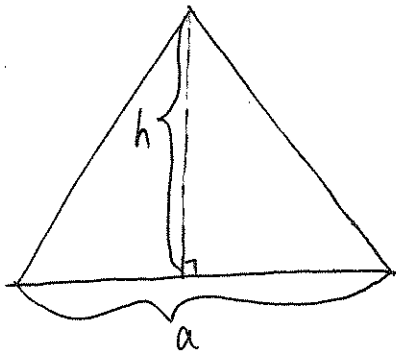
$$\Rightarrow \frac{d(x^3)}{dx} + \frac{d}{dx}(y^3) + \frac{d}{dx}(xy) = 0 \Rightarrow 0 = 3x^2 + \frac{d}{dy}(y^3) \cdot \frac{dy}{dx} + \frac{d}{dx}(x) \cdot y$$

$$+ x \cdot \frac{d}{dx}(y) = 3x^2 + 3y^2 \cdot \frac{dy}{dx} + y + x \cdot \frac{dy}{dx}$$

$$\Rightarrow -(3y^2 + x) \cdot \frac{dy}{dx} = 3x^2 + y \Rightarrow \frac{dy}{dx} = -\frac{3x^2 + y}{3y^2 + x}$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{(1,1)} = -\frac{3+1}{3+1} = -1$$

2.



height: h , base: a , area: S

1. modeling. $\frac{dh}{dt} = 1$, $\frac{dS}{dt} = 5$, $h=10$, $S=90$

$$\frac{da}{dt} = ?$$

2. equation. $S = \frac{1}{2} a \cdot h$

3. differentiation. $\frac{dS}{dt} = \frac{d}{dt} \left(\frac{1}{2} a \cdot h \right)$

$$= \frac{1}{2} \cdot \frac{da}{dt} \cdot h + \frac{1}{2} a \cdot \frac{dh}{dt}$$

$$\Rightarrow \frac{da}{dt} = \frac{\frac{dS}{dt} - \frac{1}{2} a \cdot \frac{dh}{dt}}{\frac{1}{2} \cdot h}$$

From step 1 we know: $\frac{dS}{dt} = 5$, $\frac{dh}{dt} = 1$,

$$h = 10, \text{ and } a = \frac{2S}{h} = \frac{2 \cdot 90}{10} = 18$$

$$\Rightarrow \frac{da}{dt} = \frac{5 - \frac{1}{2} \cdot 18 \cdot 1}{\frac{1}{2} \cdot 10} = \frac{5 - 9}{5} = -\frac{4}{5} \text{ (cm/min)}$$