

Quiz 6 Solution

$$1. L_2(x) = f(2) + f'(2) \cdot (x-2)$$

$$f(2) = 2^2 + 3 \cdot 2 + 1 = 11,$$

$$f'(x) = 2x + 3 \Rightarrow f'(2) = 2 \cdot 2 + 3 = 7$$

$$\Rightarrow L_2(x) = 11 + 7 \cdot (x-2) \Rightarrow L_2\left(\frac{21}{10}\right) = 11 + 7 \cdot \left(\frac{21}{10} - 2\right) \\ = 11 + 7 \cdot \frac{1}{10} = 11.7$$

2. (1) critical points:

$$\frac{dy}{dx} = \sqrt{6-x} + x \cdot \frac{1}{2} (6-x)^{-\frac{1}{2}} \cdot (-1) = \sqrt{6-x} - \frac{\frac{x}{2}}{\sqrt{6-x}} = \frac{6-x-\frac{x}{2}}{\sqrt{6-x}} \\ = \frac{6-\frac{3}{2}x}{\sqrt{6-x}}$$

$$\frac{dy}{dx} = 0 \Rightarrow 6 - \frac{3}{2}x = 0 \Rightarrow x = 6 \cdot \frac{2}{3} = 4 \in [2, 5]$$

$\frac{dy}{dx}$ DNE $\Rightarrow x=6$, but 6 is not in the domain $[2, 5]$.

So critical point is $x=4$.

(2) end points: $x=2$, $x=5$

(3) Compare the values: $y(4) = 4 \cdot \sqrt{6-4} = 4\sqrt{2}$, $y(2) = 2 \cdot \sqrt{6-2} = 4$,
 $y(5) = 5 \cdot \sqrt{6-5} = 5$.

If you don't know $4\sqrt{2}$ you can compare the squares.

$(4\sqrt{2})^2 = 16 \cdot 2 = 32$, $4^2 = 16$, $5^2 = 25 \Rightarrow 4\sqrt{2}$ is the absolute maximum ($x=4$)
4 is the absolute minimum ($x=2$)