Quiz 7. Solution

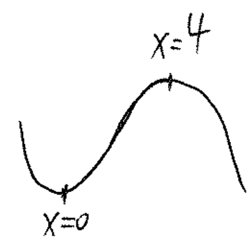
1. (a) $f'(c) = \dfrac{f(3) - f(1)}{3 - 1}$

(b) $f'(c) = \dfrac{(3+\frac{1}{3}) - (1+\frac{1}{1})}{3-1} = \dfrac{2}{3} \Rightarrow 1 - \dfrac{1}{c^2} = \dfrac{2}{3} \Rightarrow c = \pm\sqrt{3}$

since $c \in (1,3) \Rightarrow c = \sqrt{3}$

2. (a) critical points: $f'(x) = 0$ or $f'(x)$ DNE $\Rightarrow x = 0, 4, 6$
(all in definition domain)

	$4-x$	$x^{\frac{1}{3}}$	$(6-x)^{\frac{2}{3}}$	$f'(x)$	$f(x)$
$x<0$:	+	−	+	−	↓
$0<x<4$:	+	+	+	+	↑
$4<x<6$:	−	+	+	−	↓
$x>6$:	−	+	+	−	↓

$\Rightarrow$ 

$\Rightarrow$ Increasing: $x \in (0,4)$; Decreasing: $x \in (-\infty, 0) \cup (4, \infty)$

local minimum: $x = 0$, $y = f(0) = 0$; local maximum: $x = 4$, $y = f(4) = 4^{\frac{2}{3}} \cdot 2^{\frac{1}{3}} = \sqrt[3]{32}$

(b) consider the points where $f''(x) = 0$ or $f''(x)$ DNE: $x = 0, 6$

	-8	$x^{\frac{4}{3}}$	$(6-x)^{\frac{5}{3}}$	$f''(x)$	$f(x)$
$x<0$:	−	+	+	−	concave down
$0<x<6$:	−	+	+	−	concave down
$x>6$:	−	+	−	+	concave up

$\Rightarrow$ Concave up: $x \in (6, \infty)$; concave down: $x \in (-\infty, 6)$

Inflection point: $x = 6$, $y = f(6) = 0 \Rightarrow (6, 0)$. (only one inflection point.)