# Quiz 8. solution

(a) $f(x)=0 \Rightarrow x=0 \Rightarrow$ x-intercept: $(0,0)$; $f(0)=0 \Rightarrow$ y-intercept: $(0,0)$.

(b) ① $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^3}{x^2-9} = \lim\limits_{x\to\infty} \dfrac{x}{1-\frac{9}{x^2}} = \infty$, similarly $\lim\limits_{x\to-\infty} f(x) = -\infty$

$\Rightarrow$ There is no horizontal asymptote;

② Consider $\lim\limits_{x\to a} f(x) = \infty$, we may consider $x^2-9=0 \Rightarrow x=\pm 3$.

$\lim\limits_{x\to 3^+} f(x) = \infty$, $\lim\limits_{x\to 3^-} f(x) = -\infty$, $\lim\limits_{x\to -3^+} f(x) =$ [illegible], $\lim\limits_{x\to -3^-} f(x) = -\infty$.

Vertical asymptote: $x=\pm 3$;

③
$$\begin{array}{r} x+0 \\ x^2-9 \overline{\smash{)}\, x^3+0\cdot x+0} \\ \underline{x^3-9x} \\ 9x+0 \end{array}$$
$\Rightarrow$ $y=x$ is a slant asymptote.

(c) Critical points: $f'(x)=0$ or DNE $\Rightarrow \dfrac{x^2(x^2-27)}{(x^2-9)^2} = 0$ or DNE

$\Rightarrow x=0, \pm\sqrt{27}, \pm 3$, where $\pm 3$ are not in the domain.

| | $-\sqrt{27}$ | $-3$ | $0$ | $3$ | $\sqrt{27}$ | |
|---|---|---|---|---|---|---|
| $x^2-27$: | + | − | − | − | − | + |
| $\dfrac{x^2}{(x^2-9)^2}$: | + | + | + | + | + | + |
| $f'$: | + | − | − | − | − | + |

$\Rightarrow$ $f$ is increasing on: $(-\infty, -\sqrt{27}) \cup (\sqrt{27}, \infty)$

$f$ is decreasing on: $(-\sqrt{27}, -3) \cup (-3, 3) \cup (3, \sqrt{27})$

(d) Consider $f''=0$ or undefined: $\dfrac{18x\cdot(x^2+27)}{(x^2-9)^3}=0$ or undefined

$\Rightarrow$ $x=0,\ \pm 3$, where $\pm 3$ are not in the domain.

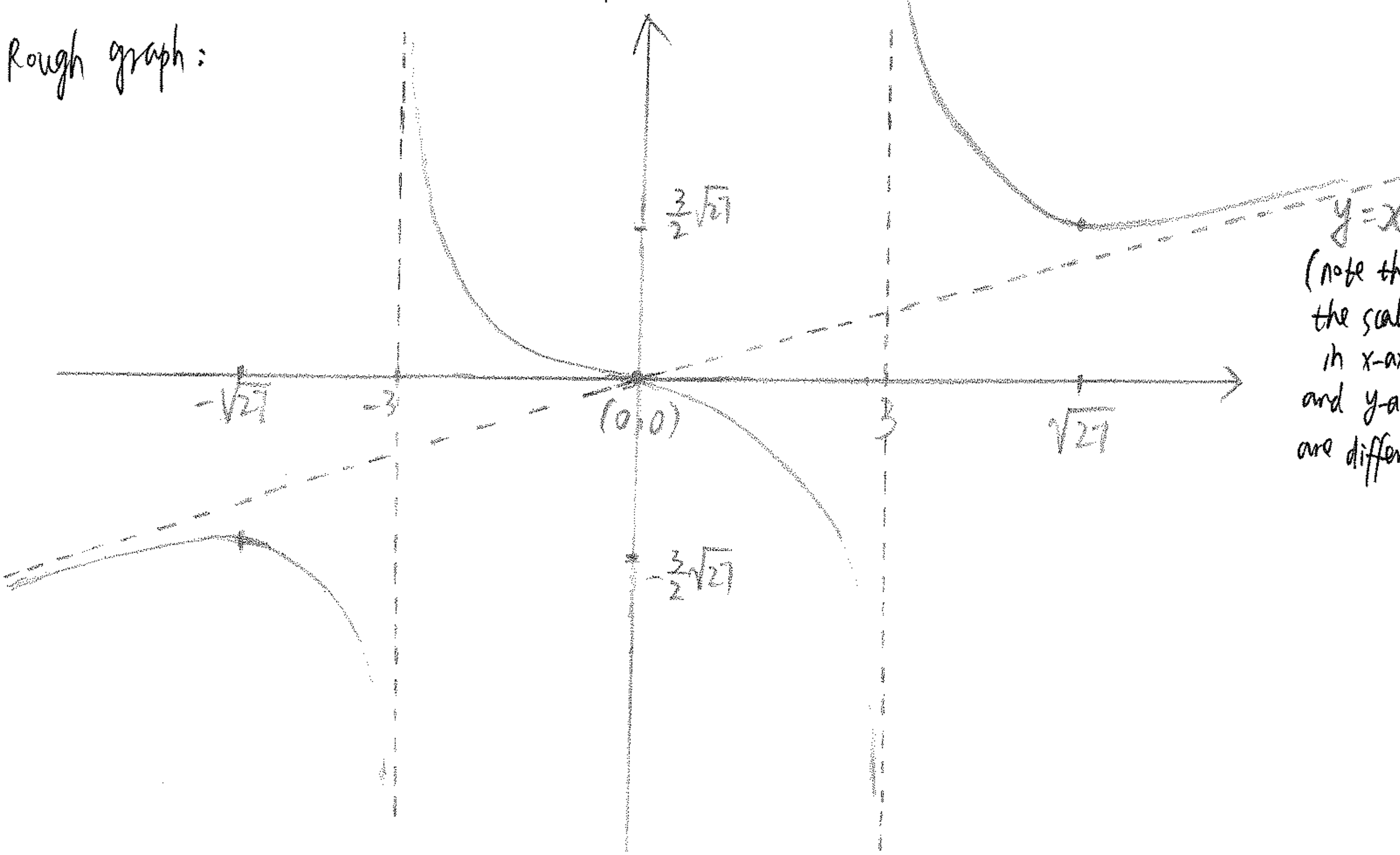
| | $(-\infty,-3)$ | $(-3,0)$ | $(0,3)$ | $(3,\infty)$ |
|---|---|---|---|---|
| $x$: | $-$ | $-$ | $+$ | $+$ |
| $(x^2-9)^3$: | $+$ | $-$ | $-$ | $+$ |
| $x^2+27$: | $+$ | $+$ | $+$ | $+$ |
| $f''$: | $-$ | $+$ | $-$ | $+$ |

$\Rightarrow$ $f$ is concave up on: $(-3,0)\cup(3,\infty)$

$f$ is concave down on: $(-\infty,-3)\cup(0,3)$

(e) Note that $f(-x)=-f(x)$, so $f(x)$ is odd and symmetric about the origin.

Rough graph:




(note that the scales in x-axis and y-axis are differen