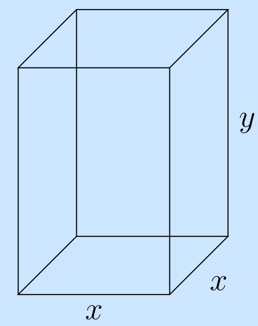
13. (20 points) If a total of 300 cm^2 of material is to be used to make a box with a square base and an open top, find the largest volume of such a box.

Solution:

Consider the picture to the right. We are trying to maximize

$$\text{Volume} = V = x^2 y$$



The restricting equation is surface area:

$$\begin{aligned} \text{Surface Area} &= 300 \\ 4xy + x^2 &= 300 \\ y &= \frac{300 - x^2}{4x} \end{aligned}$$

Giving a volume equation of one variable:

$$V(x) = \frac{1}{4}\left(300x - x^3\right) \qquad \text{with } x \in (0, 10\sqrt{3})$$

To maximize lets find critical points:

$$\begin{aligned} V'(x) &= \frac{1}{4}(300 - 3x^2) && \text{(Never undefined so set to 0.)} \\ 0&\frac{1}{4}(300 - 3x^2) \\ x^2 &= 100 \\ x &= 10 && \text{(Given domain.)} \end{aligned}$$

We verify that $x = 10$ is a max since:
Giving the largest volume of

$$V(10) = \frac{1}{4}(300(10) - (10)^3) = \frac{1}{4}(2000) = \boxed{500 \text{ cm}^3}$$