2.4. $a = \sqrt[3]{5-\sqrt{3}} \Rightarrow a^3 = 5-\sqrt{3} \Rightarrow (a^3-5)^2 = 3$

$\Rightarrow a^6 - 10a^3 + 22 = 0 \quad \cdots (*)$

Assume that $a \in \mathbb{Q}$, then a is a rational solution of $(*)$

By the rational zeros theorem $a = \frac{c}{d}$, $c/22$ and $d/1$

$\Rightarrow c = \pm 1, \pm 2, \pm 11, \pm 22$, $d = \pm 1$

$\Rightarrow a = \pm 1, \pm 2, \pm 11, \pm 22$, however, none of them are solutions of $(*)$.

So $a \notin \mathbb{Q}$

3.4. (vii) of Theorem 3.2.

$0 < a \Rightarrow 0 < a^{-1}$ by (vi) of 3.2

$0 < b \Rightarrow 0 < b^{-1}$ by (vi) of 3.2

We have to prove $b^{-1} < a^{-1}$. Assume that $a^{-1} \leq b^{-1}$.

Consider $c = ab$. $0 \leq a$ and $0 \leq b \Rightarrow 0 \leq c = ab$ by (iii) of (3.2)

$\Rightarrow (a^{-1})c \leq (b^{-1})c$ by O5.

$\Rightarrow (a^{-1})(ab) \leq (b^{-1})(ab)$, Left $= (a^{-1}a)b = b$
Right $= b^{-1}(ba) = a$

So $b \leq a$, which contradicts to $a < b$. $\Rightarrow b^{-1} < a^{-1}$.

In all, $0 < b^{-1} < a^{-1}$.

4.16. Denote $S = \{r \in \mathbb{Q} : r < a\}$

(i) $\forall s \in S$, $s < a \Rightarrow s \leq a$

(ii) $\forall M_1 < a$, by the denseness of $\mathbb{Q}$, $\exists r \in \mathbb{Q}$ such that $M_1 < r < a$.

$\Rightarrow r \in S$ and $M_1 < r$.