19.7. $f(x)$ is continuous on $[0, \infty) \Rightarrow f(x)$ is continuous on $[0, k]$

$\Rightarrow f(x)$ is uniformly continuous on $[0, k]$.

Also $f(x)$ is uniformly continuous on $[k, \infty)$

$\Rightarrow \forall \epsilon > 0, \ \exists \delta_1 > 0$, s.t. $\forall x, y \in [0, k]$ and $|x-y| < \delta_1$, we have $|f(x) - f(y)| < \frac{\epsilon}{2}$;

$\forall \epsilon > 0, \ \exists \delta_2 > 0$, s.t. $\forall x, y \in [k, \infty)$ and $|x-y| < \delta_2$, we have $|f(x) - f(y)| < \frac{\epsilon}{2}$.

$\Rightarrow \ \exists \delta = \min\{\delta_1, \delta_2\} > 0$, s.t. $\forall x, y \in [0, \infty)$ and $|x-y| < \delta$, we have:
(assume $x < y$ without loss of generality)

case 1: $0 \le x < y \le k$. Because $|x-y| < \delta \le \delta_1$, $|f(x) - f(y)| < \frac{\epsilon}{2} < \epsilon$.

case 2: $k \le x < y$. Because $|x-y| < \delta \le \delta_2$, $|f(x) - f(y)| < \frac{\epsilon}{2} < \epsilon$.

case 3: $0 \le x \le k \le y$. $|x-k| \le |x-y| < \delta \le \delta_1$ and $|y-k| \le |x-y| < \delta \le \delta_2$

$\Rightarrow |f(x) - f(k)| < \frac{\epsilon}{2}$ and $|f(y) - f(k)| < \frac{\epsilon}{2} \Rightarrow |f(x) - f(y)| = |f(x) - f(k) + f(k) - f(y)|$

$\le |f(x) - f(k)| + |f(y) - f(k)| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$

$\Rightarrow f(x)$ is uniformly continuous on $[0, \infty)$.

20.16. (a) Otherwise $L_1 > L_2$, consider $\epsilon = \frac{L_1 - L_2}{2} > 0$.

$\exists \delta_1 > 0$, s.t. $\forall x \in (a, a+\delta_1)$, $|f_1(x) - L_1| < \epsilon \Rightarrow f_1(x) > L_1 - \epsilon = \frac{L_1 + L_2}{2}$

$\exists \delta_2 > 0$, s.t. $\forall x \in (a, a+\delta_2)$, $|f_2(x) - L_2| < \epsilon \Rightarrow f_2(x) < L_2 + \epsilon = \frac{L_1 + L_2}{2}$.

Now take $x_0 \in (a, a + \min\{\delta_1, \delta_2\}) \subseteq (a, a+\delta_1) \cap (a, a+\delta_2)$, we have

$f_1(x_0) > \frac{L_1 + L_2}{2} > f_2(x_0)$ which contradicts to $f_1(x) \le f_2(x)$, $\forall x \in (a, b)$

(b) $f_1(x) = 1 - (x-a)$, $f_2(x) = 1 + (x-a)$

$f_1(x) < f_2(x)$, $\forall x \in (a, b)$. But $\lim_{x \to a^+} f_1(x) = \lim_{x \to a^+} f_2(x) = 1$.

24.10. (a) $f_n \to f$ uniformly on S

$\Rightarrow \forall \epsilon > 0$, $\exists N_1 > 0$, s.t. $\forall n > N_1$ and $\forall x \in S$, we have $|f_n(x) - f(x)| < \frac{\epsilon}{2}$

$g_n \to g$ uniformly on $S \Rightarrow \forall \epsilon > 0$, $\exists N_2 > 0$, s.t. $\forall n > N_2$ and $\forall x \in S$, we have $|g_n(x) - g(x)| < \frac{\epsilon}{2}$.

$\Rightarrow \exists N = \max\{N_1, N_2\} > 0$, s.t. $\forall n > N$ and $\forall x \in S$, we have

$$\left| f_n(x) + g_n(x) - (f(x) + g(x)) \right| \le |f_n(x) - f(x)| + |g_n(x) - g(x)| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$$

$\Rightarrow f_n + g_n \to f + g$ uniformly on S.

(b) ~~the~~ According to 24.11 (b), the uniform convergence is not necessarily true for products.