Supplemental Exercises for Section 2.4

The formula $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$ can be made more useful by observing that if $\lim_{x\to a} f(x) = 0$ and $f(x)$ is never 0, then $\lim_{x\to a} \dfrac{\sin f(x)}{f(x)} = 1$. For example

1. $\displaystyle\lim_{x\to 0} \frac{\sin x^2}{x^2} = 1.$

2. $\displaystyle\lim_{x\to 0} \frac{\sin x^2}{x} = \lim_{x\to 0} \frac{\sin x^2}{x^2} \lim_{x\to 0} \frac{x^2}{x} = 1 \cdot 0 = 0.$

3. $\displaystyle\lim_{x\to -1} \frac{\sin(x^2 - x - 2)}{x+1} = \lim_{x\to -1} \frac{\sin(x^2 - x - 2)}{(x^2 - x - 2)} \lim_{x\to -1} \frac{(x^2 - x - 2)}{x+1}$
$\displaystyle = 1 \cdot \lim_{x\to -1} \frac{(x+1)(x-2)}{x+1} = -3.$

4. $\displaystyle\lim_{x\to 1} \frac{\sin(1-\sqrt{x})}{x-1} = \lim_{x\to 1} \frac{\sin(1-\sqrt{x})}{1-\sqrt{x}} \frac{1-\sqrt{x}}{x-1} = 1 \cdot \lim_{x\to 1} \frac{(1-\sqrt{x})(1+\sqrt{x})}{(1+x)(1+\sqrt{x})} = .$
$\displaystyle\lim_{x\to 1} \frac{1-x}{(x-1)(1+\sqrt{x})} = -\frac{1}{2}.$

Find each of the following limits.

5. $\displaystyle\lim_{x\to 0} \frac{\sin(1-\cos x)}{x}$

6. $\displaystyle\lim_{x\to 0^+} \frac{\sin x}{\sin \sqrt{x}}$

7. $\displaystyle\lim_{x\to 0} \frac{\sin(\sin x)}{x}$

8. $\displaystyle\lim_{x\to 0} \frac{\sin(x^2 + x)}{x}$

9. $\displaystyle\lim_{x\to 2} \frac{\sin(x^2 - 4)}{x-2}$

10. $\displaystyle\lim_{x\to 9} \frac{\sin(\sqrt{x} - 3)}{x-9}$

In addition compute the following limit.

11. $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{x^2}.$

Selected Answers

2.	0
4.	1
5.	0
8.	1
11.	$\frac{1}{2}$.